Analytical Assignment-2
Solutions

1. All words at a Hamming distance of 1 from a given codeword (in the M-of-N code) have either M+1 or M-1 bits that are 1 are therefore, at least 2. Changing one bit in a codeword from 0 to 1 and another bit from 1 to 0 will generate a valid codeword at a Hamming distance of 2.

2.

Data word	0000	0001	0010	0011	0100	0101	0110	0111
Non-Separable	00000	00011	00110	00101	01100	01111	01010	01001
Separable	00000	00011	00101	00110	01001	01010	01100	01111

Data word	1000	1001	1010	1011	1100	1101	1110	1111
Non-Separable	11000	11011	11110	11101	10100	10111	10010	10001
Separable	10001	10010	10100	10111	11000	11011	11101	11110

3. (a) The received message is

$$P(x) = (x) + E(x)$$

where $C(x)$ and $E(x)$ are the transmitted codeword and the error polynomial respectively. If a single bit error in position i occurs then $E(x) = x^i$. If $G(x)$ has more than one term, it can not divide x^i and thus, all single bit errors will be detected.

(b) For double bit errors,

$$E(x) = x^i + x^j = x^i(1 + x^{(j-i)}).$$

For double bit error to be detected, neither x^i nor $(1 + x^{(j-i)})$ should be divisible by $G(x)$. This holds if $G(x)$ has a factor with 3 terms.

(c) Assume that $E(x)$ contains an odd number of terms and is divisible by $x+1$.

Then, $E(x) = (x+1)\,B(x)$. Thus $E(1) = 0$ which means that $E(x)$ must contain an even number of terms. Both CRCs can detect all single, double and odd numbers of bit errors plus all bursts upto 16 bit errors.