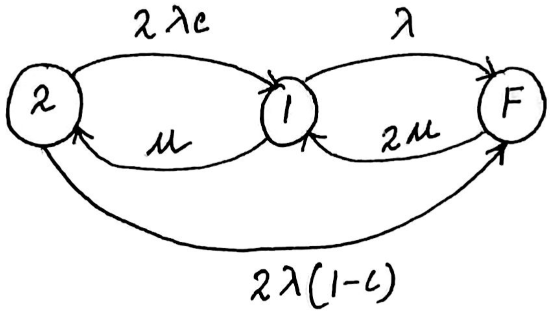
Analytical Assignment - 1
Solutions

1. ■ Markov model for the system:



■ For the long term availability, we write the steady-state balance equations.

$$2\lambda P_2 = \mu P_1$$
$$2\lambda(1-c)P_2 + \lambda P_1 = 2\mu P_F$$
$$P_2 + P_1 + P_F = 1$$

For convenience, denote $\rho = \lambda/\mu$. The solution to the above expression -

$$P_2 = \frac{1}{1+\rho(3-c)+\rho^2}$$

$$P_1 = \frac{2\rho}{1+\rho(3-c)+\rho^2}$$

and the availability is,

$$P_1 + P_2 = \frac{1+2\rho}{1+\rho(3-c)+\rho^2}$$

If $\lambda = 2\mu$, $\rho = 0.5$,

$$\text{Availability} = \frac{2}{2.75-0.5c} = \frac{8}{11-2c}$$

2. Let the state of the system be the number of modules that are still functional and let T_K be the time spent in state K. Then $E[T_K] = \frac{1}{K\lambda}$.

Since $MTTF = \sum_{K=1}^{N} E[T_K]$, the result follows immediately.

<u>Another way:</u>

The reliability of the parallel system is,

$$R(t) = 1-(1-e^{-\lambda t})^N$$

Denoting $x = (1-e^{-\lambda t})$, the above equation can be rewritten as,

$$R(t) = (1-x)\left(1+\sum_{K=1}^{N-1} x^K\right)$$

The MTTF is calculated from $\int_0^\infty R(t)\,dt$.

Since $dx = \lambda e^{-\lambda t} dt$, we obtain,

$$MTTF = \frac{1}{\lambda}\int_0^1 \sum_{K=0}^{N-1} x^K dx$$

$$= \frac{1}{\lambda}\left[\sum_{K=0}^{N-1} \frac{x^{K+1}}{K+1}\right]_0^1$$

$$= \frac{1}{\lambda}\sum_{K=0}^{N-1} \frac{1}{K+1}$$

$$= \frac{1}{\lambda}\sum_{K=1}^{N} \frac{1}{K}$$