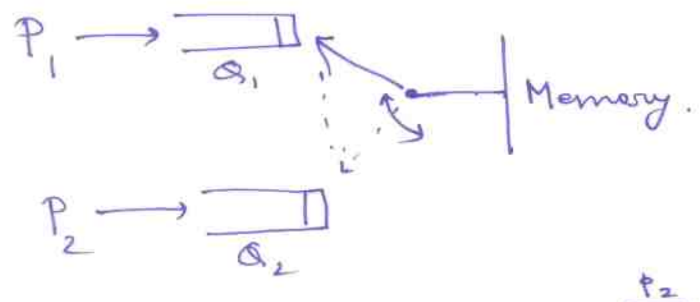
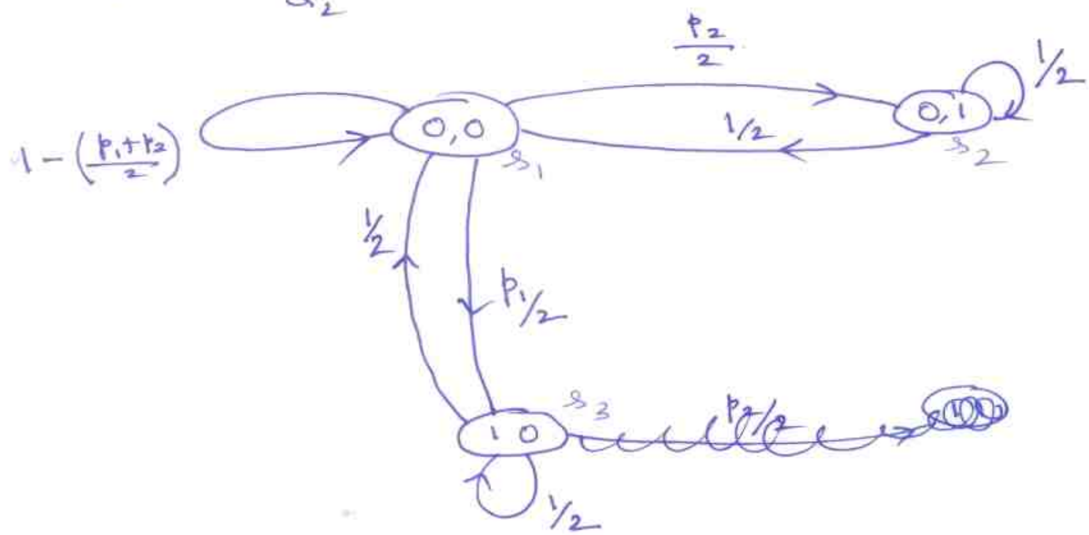
1. Each queue Q_i shall contain at most 1 pending request, $(i = 1, 2)$.



States, $\langle 0,0 \rangle$
$\langle 0,1 \rangle$
$\langle 1,0 \rangle$
$\langle 1,1 \rangle$



$$\delta(s_1, s_2) = P(\text{switch at position 1 while mem. req. generated for } P_2) = \frac{p_2}{2}.$$

Similarly, $\delta(s_1, s_3) = \frac{p_1}{2}$.

By DTMC property, $\delta(s_1, s_1) = 1 - \frac{p_1 + p_2}{2}$

Physical explanation: →

$$\delta(s_1, s_1) = P(\text{switch at position 1 \& no req. generated at } P_2)$$
$$+ P(\text{- - - - - - 2 \& no req. generated at } P_1)$$
$$= \frac{1}{2}\cdot(1 - p_2) + \frac{1}{2}\cdot(1 - p_1)$$
$$= 1 - \left(\frac{p_1 + p_2}{2}\right)$$

$$\delta(s_3, s_1) = P(\text{switch is at position } 1) = \frac{1}{2}$$
$$\delta(s_2, s_1) = P(\text{" " " " } 2) = \frac{1}{2}$$
$$\delta(s_2, s_2) = P(\text{" " " " } 1) = \frac{1}{2}$$
$$P(s_3, s_3) = P(\text{" " " " } 2) = \frac{1}{2}$$

Why ??

Next part, i.e., computing steady state probabilities is straight fwd.

2.

TMR has higher reliability w.r.t. simplex for short mission times.

⟶ done in a) Koren & Krishna
b) K. Trivedi. ⟶ gives a more analytical soln.

TMR-simplex (Trivedi's book)

$X, Y, Z \longrightarrow$ time to failure of 3 components (λ). assume.
$W \longrightarrow$ residual " " " of surviving component. (λ)
$L \longrightarrow$ overall " " " of TMR-simplex.

$$L = \min(X, Y, Z) + W$$

$\min(X,Y,Z)$ ∴ exponentially distributed with parameter (3λ)
⟶ follows from order statistics of mutually independent, identically distributed continuous random variables.

~~to the~~ W follows distribution with parameter λ due to memoryless property of exponential distribution.

L has a 2 stage hypoexponential distribution with parameters $\underset{(\lambda_2)}{3\lambda}$ & $\underset{(\lambda_1)}{\lambda}$. $[\lambda_2 > \lambda_1]$

∴ the distribution function,

$$F_L(t) = 1 - \frac{\lambda_2}{\lambda_2 - \lambda_1} \cdot e^{-\lambda_1 t} + \frac{\lambda_1}{\lambda_2 - \lambda_1} \cdot e^{-\lambda_2 t}$$

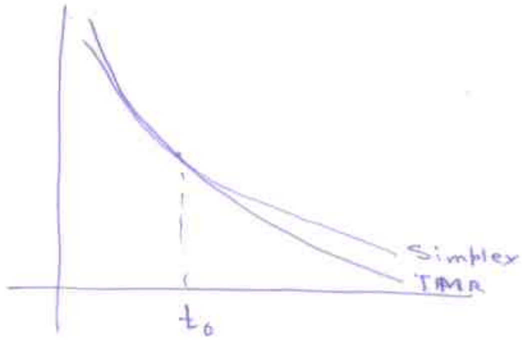
$$= 1 - \frac{3\lambda}{2\lambda} \cdot e^{-\lambda t} + \frac{\lambda}{2\lambda} \cdot e^{-3\lambda t}$$

$$= 1 + \frac{e^{-3\lambda t}}{2} - \frac{3}{2} \cdot e^{-\lambda t}.$$

$$R(t) = 1 - F_L(t) = \frac{3}{2} \cdot e^{-\lambda t} - \frac{1}{2} \cdot e^{-3\lambda t}$$

2)

$$R_{TMR} = 3R^2(t) - 2R^3(t)$$
$$= 3e^{-2\lambda t} - 2e^{-3\lambda t}$$



Simplex $\rightarrow e^{-\lambda t}$.

Solving for t_0,

$$e^{-\lambda t_0} = 3e^{-2\lambda t_0} - 2e^{-3\lambda t_0}$$

$$\Rightarrow t_0 = \frac{0.7}{\lambda}$$

Reliability of TMR is higher upto t_0.

$$MTTF_{TMR} = \frac{5}{6\lambda} < \frac{1}{\lambda}\,(MTTF_{simplex}) < \frac{4}{3\lambda}\,(MTTF_{TMR\,simplex})$$

3) a)

$$R_{parallel}(t) = 1 - (1-e^{-\lambda t})^N$$

with $x = (1-e^{-\lambda t})$,

~~R(t) = (...)~~

$$dt = \frac{1}{\lambda} \cdot \frac{dx}{1-x} \longrightarrow$$

$$MTTF = \int_0^\infty R_{par}(t)\,dt$$
$$= \int_0^\infty \left[1-(1-e^{-\lambda t})^N\right] dt$$
$$= \frac{1}{\lambda}\int_0^1 \frac{1-x^N}{1-x}\,dx$$
$$= \frac{1}{\lambda}\int_0^1 \left(\sum_{i=1}^{N} x^{i-1}\right) dx$$
$$= \frac{1}{\lambda}\sum_{i=1}^{N}\int_0^1 x^{i-1}\,dx$$
$$= \frac{1}{\lambda}\sum_{i=1}^{N}\left[\frac{x^i}{i}\right]_0^1 = \frac{1}{\lambda}\sum_{i=1}^{N}\frac{1}{i}$$

3) a) discussed in class.

b) " " ". Pick up a failure of (i,j)
i-th switch in j-th stage. Reason out the resulting failure can be tolerated.

4) a) Pick up any two failures & reason out the lack of tolerance. [some i/p & o/p pair gets disconnected]

b) Associativity of cache has big impact on the performance degradation due to cache lines becoming faulty. In case of direct mapped cache; ~~the~~ all mem. blocks that get mapped to defective cache lines/blocks get excluded from any caching mechanism.

50% marks just for this → The degradation is linear w.r.t. fraction of faulty block.

In a set associative cache, the degradation is sub-linear. If one block among M-way in the same set is defective, the remaining (M-1) healthy blocks are still able to accommodate the memory blocks getting mapped to the set. However, the replacement rate for a faulty set shall increase due to higher probability of conflict misses due to decreasing no. of useful 'ways' in the set.

[~~important b~~ potentially important blocks getting replaced ~~due to~~ by the replacement method]

In fully associative ~~cache~~, any mem. block can reside in any cache block.

Degradation solely depends on probability of conflict miss.

That is why, spare caches are usually fully associative.

5.

i) A single module can → a) be working fine
b) stuck at '0'. (λ_0)
c) " " '1'. (λ_1)

Prob (a) at t
$$= e^{-\lambda_1 t} \cdot e^{-\lambda_0 t} = e^{-(\lambda_1+\lambda_0)t}$$

$$\text{Prob (b or c)} = 1 - e^{-(\lambda_1+\lambda_2)t}$$

b & c are disjoint.

$$\text{Prob (b)} = \frac{\lambda_0}{\lambda_1+\lambda_0}\left(1 - e^{-(\lambda_1+\lambda_0)t}\right)$$

ii) The result is incorrect if atleast 2 modules are stuck at 1.

Prob. of this happening for one module
$$= q(t) = \frac{\lambda_1}{\lambda_0+\lambda_1}\left(1 - e^{-(\lambda_1+\lambda_0)t}\right)$$

∴ Overall prob. of incorrect result
$$= q^3(t) + 3q^2(t)\left(1 - q(t)\right)$$

6) a) Easy one.

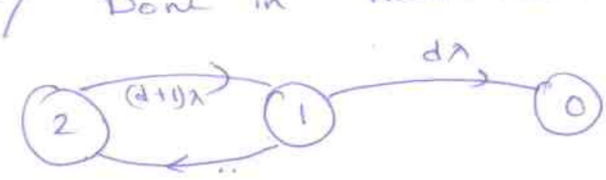
b) All words at a Hamming distance of 1 from a given codeword (in the M of N code) have either M+1 or M-1 bits that are 1. These cannot be valid codes. We start getting valid codes from distance 2 (& more).

Therefore, the Hamming distance of an M-of-N code is 2.

[The overall Hamming distance of a code is defined as such least distance.].

Take a valid code, flip two bits, one from 0 to 1 & another from 1 to 0. You get a valid code at the distance of 2.

7) Done in Koren'n'Krishna.



$$\text{MTTDL} = \frac{(2d+1)\lambda + \mu}{d(d+1)\lambda^2}$$