Formal Language And Automata Theory (CS21204)
Spring 2024

Problem sets

For $\Sigma = \{0,1\}$:

1. Design a CFG for:

$L = \{w \mid w$ contains at least three $0's\}$. w contains at least 3 0's.

Ans:

$S \rightarrow S1S1S1S;$
$S \rightarrow 1S \mid 0S \mid \epsilon$

2. Design a CFG for: $L = \{w \mid w$ contains more 1's than $0's\}$.

Ans:

$S \rightarrow S_1 \mid S_1$
$S_1 \rightarrow 0S_1 1 \mid 1S_1 0 \mid S_1 S_1 \mid 1S_1 \mid \epsilon$

3. Grammar for $L = \{w \mid w$ starts and ends with same symbol $\}$.

Ans: This actually is a regular language

$S \rightarrow 0T \mid 1U$
$T \rightarrow 0T \mid 1T \mid 0$
$U \rightarrow 0U \mid 1U \mid 1$

4. Design a CFG for: $L = \{w \mid w$ length is odd$\}$.

$S \rightarrow 0S0 \mid 0S1 \mid 1S0 \mid 1S1 \mid 0 \mid 1$

5. Similar: w is odd length with mid symbol 0

6. Similar: w is palindrome.

7. Consider the following grammar.

$$S \rightarrow aSb \,|\, bY \,|\, Ya$$
$$Y \rightarrow bY \,|\, aY \,|\, \epsilon$$

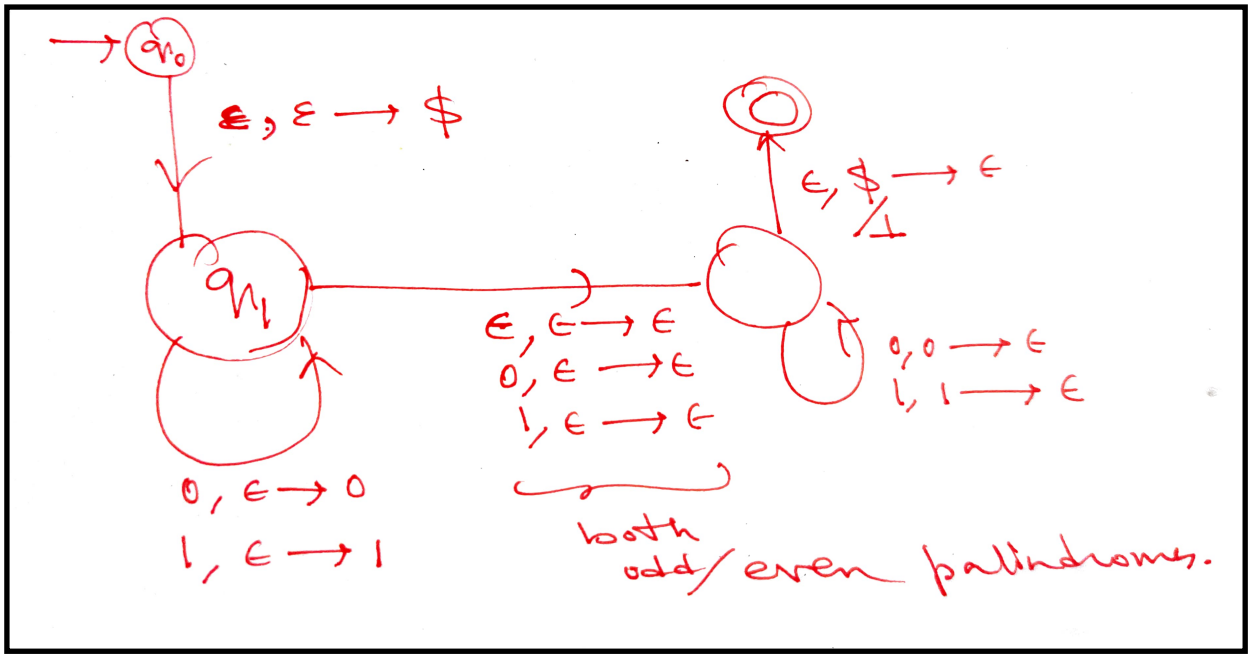
Describe in english language the language for the given grammar.

Ans: The grammar generates two kinds of strings

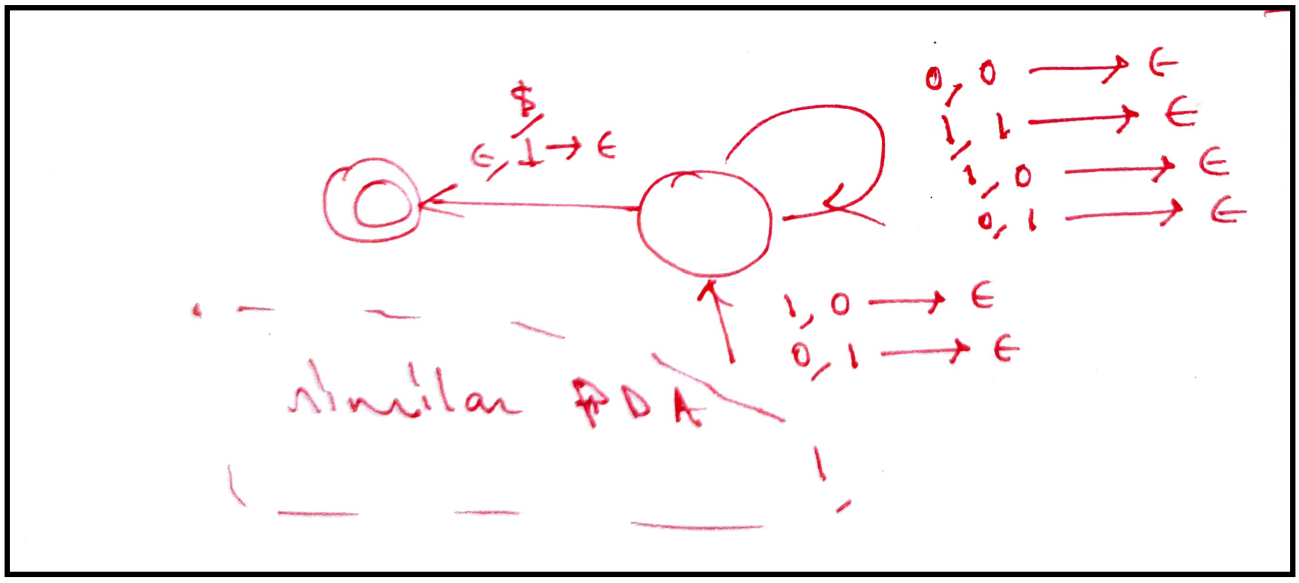
Either $a^n b(a+b)^* b^n \quad OR \quad a^n (a+b)^* a b^n$ with $n \geq 0$

Essentially this is $\Sigma^* \setminus \{a^n b^n \,|\, n \geq 0\}$

8. PDA for palindromes:



9. PDA for non-palindromes:



10. Let us define $A/B = \{w \mid wx \in A \text{ for some } x \in B\}$. If A is a CFL and B is regular, prove that A/B is a CFL

Proof idea: Let A have a corresponding PDA M and B have a DFA N.

a. Construct PDA X which accepts $A \cap B$ by parallel composition of machines M and N.
b. Any transition of X of the form $(s,t) \xrightarrow{a, A \to B} (s',t')$ is replaced by $(s,t) \xrightarrow{\epsilon, A \to B} (s',t')$.
c. Let start state of X be q_0. For all states q of A, the transition relation $\delta(q, \epsilon, A)$ is updated as $(q_0, A) \cup \{\delta(q, \epsilon, A)\}$. Essentially a nondeterministic transition is added from any state in A to q_0.
d. The above construction ensures that after simulation of w in A, a nondeterministic jump is possible to X and X makes a nondeterministic guess of x to reach final state.

With overall acceptance defined as final states of X, the overall PDA with components A (modified) and X accepts A/B. Hence this is CFL.

11. For a CFG in CNF form, prove that a string of length n can be derived in at most 2n-1 derivation steps.