Backhaul-Aware Storage Allocation and Pricing Mechanism for RSU-Based Caching Networks

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***Abstract*—Remarkable prevalence of in-car entertainment systems empowers vehicular users to download multimedia-enabled contents in transit and creates a new business opportunity for content providers (CPs). However, the timely delivery of requested content is a major concern for CPs to improve the quality of service (QoS) of their subscribe users. Roadside unit (RSU)-based caching appears as a promising solution for CPs wherein CPs proactively store their content at the RSU to reduce the content delivery time. Since the RSUs are enabled with limited storage capacity, the competition among multiple CPs for storage space is unavoidable. Further, the CPs are connected with RSUs using capacity limited backhaul links. Hence the allocation of RSUs' storage among CPs becomes a fundamental issue in RSU-based caching networks. In this paper, we design a market scenario in which the set of CPs competes for the storage space of RSUs. In the unavailability of utility and cost functions of the CPs and the RSUs, we introduce a market maker to manage the marketplace. Further, we employ iteration-based double-sided auction mechanism to compute the optimal storage allocation and corresponding payment transfer for CPs which maximizes the social welfare of the networks. The simulation results demonstrate the proposed auction mechanism improves the social welfare of the network by at least 29.3% compared to the benchmark schemes. Further, with the help of both analytical and numerical analysis, we show that the proposed auction mechanism also holds vital economical properties.**

***Index Terms*—vehicular caching, auction theory, incentive mechanism, hidden information, distributed optimization.**

I. Introduction

Recently, the demand of multimedia-enabled contents is dramatically increasing among vehicular users due to the emergence of the in-car entertainment system [1]. Such demand creates a new business opportunity for emerging vehicular content providers (CPs), including siriusxm, Twine4Car, Roku, and many more. The CP collaborates with cellular network providers, due to the ubiquitous coverage of cellular network infrastructure, for on-demand content delivery wherein the requested contents are transmitted to the vehicular users via cellular networks [2]. However, such on-demand content delivery is not an ultimate solution for CP as the current cellular network is highly congested and results in higher delay in content delivery [3]–[5]. Thus, CPs aiming to reduce the content access delay of their vehicular users must find alternate approaches. The cache-assisted content delivery (CACD) appears one of the promising approach [1], [6].

In the CACD approach, the CPs proactively store the common-request contents closer to the vehicular users, e.g., at cache-enabled RSUs. Specifically, the CACD approach consists of two phases namely *placement phase* and *delivery phase* [7]. During the placement phase, the CPs connect with RSUs via backhaul links and cache their contents in the allocated storage space of RSUs. Further, during the delivery phase, the CPs deliver the cached content to their vehicular users. The CACD approach reduces the CPs' redundant content transmission over the cellular network and also decreases the average content access delay of the vehicular users. In particular, the benefit of CPs from content caching mainly depends on the size of allocated storage and the content delivery potential of the RSU. Unfortunately, the RSUs are enabled with limited storage capacity [4] and hence unable to cache contents of all the CPs. This leads to the competition among CPs for storage space of the RSUs in CACD approach. Hence, to realize the full benefits of CACD approach it is imperative to design a mechanism which allocates storage space of RSUs among CPs for content caching.

There exists a number of works which have paid attention to the content caching in CACD approach [1], [3], [4], [8], [9]. The existing literature focus to optimize the performance of CCAD approach in terms of content access delay [1], [4], [8], availability of contents [1], [9], and network throughput [3]. These works presume that the CPs and RSUs have social bonding and the storage allocated to CPs from each RSUs are known. However, CPs and RSUs' owners are generally different entities and have self-centric goals. Specifically, RSU owners show reluctance to offer their storage space to CPs for content caching without proper incentives. Based on this fact Mangili *et al.* [10] investigated the storage allocation problem in the presence of single CP and various cache-enabled access points and uses auction theory to lease the storage space of access point. Recently, Hu *et al.* [11] proposed an auction theory-based for storage allocation in the presence of multiple CPs and RSUs. Although the auction-based countermeasures motivate RSUs to allocate their storage space to participating CPs for content caching, the capacity of backhaul links between CPs and RSUs are overlooked. The backhaul links are utilized during the placement phase in RSU-based caching.

The above discussions motivate us to design a mechanism for RSU-based caching system which enables interaction among CPs and RSUs for storage allocation and incentive transfer while considering their backhaul link capacity. Specifically, the proposed mechanism computes the amount of storage allocated to each CP from each RSU and corresponding incentive transfer. The mechanism also maximizes the social welfare of the RSU-based caching system while accounting the fact that the utility function of CPs and the cost function of RSUs are their private information. Technically, We are concerned about the following set of questions: *i) How much*

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storage space each RSU should allocate to the participating CPs, ii) How much incentive each CP should transfer to RSUs, for the benefit of allocated storage space?

II. Related Work

RSU-based caching in vehicular networks has received research attention in recent times to optimize the performance of networks in terms of throughput [3], content access delay [1], [4], [8], and availability of contents [1], [9]. Malandrino *et al.* [3] discussed the impact of the RSU-based caching on vehicular network throughput. The authors revealed that the planned deployments of the caching unit double the throughput compared to the random placement. Su *et al.* [1] proposed an edge caching scheme wherein multiple road side units (RSUs) dedicate their storage space for content caching. A moving vehicle downloads the requested content directly from connected RSU, if the RSU has a replica of that content. Otherwise, the RSU provides the requested content to the vehicle by fetching content from nearby RSUs or from the remote server. The authors obtained the content placement policy to minimize the content access delay. To further improve the content access delay Kazmi *et al.* [4] considered multi-access edge computing-enabled RSUs which can compress the content before transmitting to vehicular users. Bitaghsir *et al.* [9] discussed the role of the mobile cache unit and obtained its optimal path based on the social degree of vehicular users to maximize the availability of content in the local cache. The aforementioned efforts underline the comprehensive feature of RSU-based caching in the vehicular network. However, these works are limited to the scenarios where the deployed RSUs are owned by the CP itself or by the cooperative third party entities. Unfortunately, in general, RSUs are owned by rational and self-centered entities and do not have any incentive to cache the content of CPs. Hence, we need an incentive mechanism that motivates RSUs' owners to participate in content caching. To deal with the rational entities, a game theory-based incentive mechanism has been investigated in a few recent works [10]–[12].

Zhou *et al.* [12] proposed a vehicular edge computing framework wherein the vehicles lease their idle storage and computation resources to the base station (BS). The BS assigns that leased resources to its users for task execution. Since the information regarding the resources are vehicles specific and are private in nature, the authors proposed a contract theory-based mechanism for BS. Additionally, Mangili *et al.* [10] investigated the storage allocation problem in the presence of a single CP and various cache-enabled access point, and employed auction theory to lease the storage space of the access point. Recently, Hu *et al.* [11] proposed an auction theory-based scheme for storage allocation in the presence of multiple CPs and RSUs. Although the existing countermeasures motivate the RSUs to allocate their storage space to participating CPs for content caching, the capacity of backhaul links between CPs and RSUs are overlooked which are utilized during the placement phase in RSU-based caching.

In literature, auction-based mechanism has been employed for various resource allocation problems, such as, bandwidth allocation [13], data offloading [14], and task offloading [15]. Specifically, Kelly *et al.* [13] proposed a Walrasian auction-based bandwidth allocation scheme wherein the users are modeled as buyers and the single network operator is modeled as seller. The buyers compute their demand for the bandwidth resources at every possible price and submit the same to the auctioneer. In the proposed scheme, the auctioneer computes the optimal bandwidth allocation among the users while considering the total available of the bandwidth. However, the proposed scheme is limited to scenarios involving a single seller and multiple buyers. Further, Iosifidis *et al.* [14] extended the aforementioned scheme while considering the case with multiple sellers and multiple buyers. Specifically, the authors studied the data offloading issues in the presence of multiple network operators and access points, which are modeled as buyers and sellers, respectively. Further, the authors considered the offloading capacity of the access points (i.e., the sellers) for computing the optimal data offloading. The RSU-based storage space allocation problem, formulated in this work, is closely related to the problems formulated in [13], [14]. However, in our system model, there is an additional constraint due to the backhaul link connecting the CPs and RSUs. This plays a significant role in the optimal storage allocation in RSU-based caching system. Thus, we propose a double auction-based mechanism for RSU-based caching system which enables interaction among the CPs and RSUs for storage allocation and incentive transfer while considering their backhaul link capacity.

In particular, our contributions are as follows:

- We conceptualize a storage space trading market scenario in which multiple CPs compete to lease the storage of RSUs for caching their popular contents. Based on the conflicting utility and cost functions of participating entities, namely CPs and RSUs, we formulate an optimization problem to maximize the social welfare.
- We proposed an iteration-based double-sided auction mechanism to coordinate the storage space trading among the CPs and RSUs. The proposed mechanism does not need to have any prior knowledge of the utility and cost functions of the participating CPs and RSUs. The self-centric behavior of the participating entities is also considered in the proposed mechanism.
- Further, we design pricing and reimbursement policies that motivate the CPs and the RSUs to signal their storage requirement and willingness truthfully. The proposed policies and the alternative optimization are used to obtain the optimal allocation which maximizes the social welfare.
- We also analytically prove that the proposed auction mechanism is individually rational, budget balanced, and incentive compatible. Finally, through extensive numerical simulation, we demonstrated the efficacy of the proposed mechanism.

III. System Model

We consider a RSU-based caching system wherein the set of CPs utilizes the storage space of RSUs to cache their content, as illustrated in Fig. 1. Let $\mathcal{C} = \{1, 2, \ldots C\}$ and

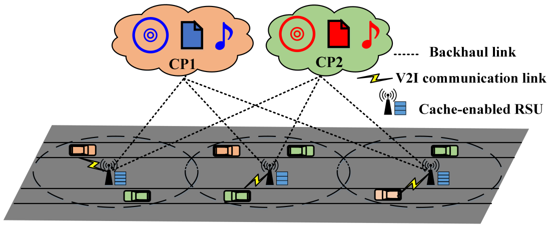


Fig. 1: RSU-based caching system with multiple CPs and RSUs

$\mathcal{R} = \{1, 2, \dots R\}$ denote the set of CPs and RSUs, respectively. The CP $c \in \mathcal{C}$ utilizes its backhaul link l_{cr} to transmit content to the RSU $r \in \mathcal{R}$ during the placement phase. Let b_{cr} is the capacity of backhaul link l_{cr} which is defined as the maximum amount of content (in GB) CP c can transmit to RSU r during the placement phase [4]. Further, each RSU $r \in \mathcal{R}$ has a limited storage capacity of S_r.

Each CP serves the content requests of its subscribed vehicular users. Specifically, on the arrival of content request the CP checks the availability of content in the nearby RSUs and serves the requested content using vehicular-to-infrastructure (V2I) communication link. In the case of nonexistence, the CP serves the requested content via a cellular link, which results in high content delivery time. To minimize the content delivery time, each CP motivates the RSUs to store its respective contents. Since each RSU $r \in \mathcal{R}$ has a limited storage capacity of S_r, we model the situation between CPs and RSUs as a market wherein the CPs act as buyers and RSUs act as sellers of the available storage capacity. To this end, we discuss the utility function of the CPs and the cost function of the RSUs.

A. Utility Model of CP

Each CP obtains certain satisfaction (utility) by providing uninterrupted content delivery to its subscribed users. For that, each CP requests storage space from RSUs to store its content. The benefit of CP from caching at particular RSU depends on the amount of storage space allocated and the set of its contents stored. We assume that the subscribed users have a homogeneous content preference and the CP stores contents according to their popularity in the allocated space. Further, each CP has a preferences towards each RSU which depends on the various factors such as no. of subscribed users access the particular RSU, their subscription plan, and so on. Therefore, the caching benefit of CP depends on the requested amount of RSU's storage space and its preference.

We model the utility of the CP c corresponding to RSU r using bi-dimensional function $\mathcal{U}_{cr}(\theta_{cr}, x_{cr})$ where x_{cr} denotes the requested amount of storage space and the preference of CP $c \in \mathcal{C}$ to RSU $r \in \mathcal{R}$ and $\theta_{cr} \in [0, 1]$ signifies the preference of the CP. Here, $\theta_{cr} = 0$ indicates that CP c is not interested to cache its content at RSU r. We assume that the utility $\mathcal{U}_{cr}(\theta_{cr}, x_{cr})$ is a strictly concave, positive, and increasing function of x_{cr}. The concavity of the function satisfies the law of diminishing marginal utility of the CP for successive units of storage space. The utility function captures the fact that the benefit of CP c not only depends upon total requested amount of storage space $X_c = \sum_{r \in \mathcal{R}} x_{cr}$ but also depends on the storage request vector of the CP ($\boldsymbol{x_c} = (x_{c1}, x_{c2}, \dots x_{cR})$), i.e., the amount of storage space requested from individual RSU. Further, we assume that the utility $\mathcal{U}_{cr}(\theta_{cr}, x_{cr})$ is an increasing function of θ_{cr}, as it is beneficial for the CP to cache contents at the preferable RSU. Specifically, we define the utility of CP over all the RSUs as follows:

$$\mathcal{U}_c(\boldsymbol{x_c}, \boldsymbol{\theta}_c) = \sum_{r \in \mathcal{R}} \mathcal{U}_{cr}(\theta_{cr}, x_{cr}) \tag{1}$$

where $\boldsymbol{\theta}_c = (\theta_{cr})_{r \in \mathcal{R}}$.

The CPs utilize the connecting backhaul link during the placement phase to store their contents in the storage space of the RSUs. Therefore, the total amount of content cached by the CPs during the placement phase is bounded by the maximum capacity (in GB) [16]. Let $b_{cr} \geq 0$ be the maximum capacity of the link connecting CP c and RSU r. Therefore, the amount of storage space requested (x_{cr}) by the CP c is bounded by b_{cr}, i.e.,

$$x_{cr} \leq b_{cr} \quad \forall c \in \mathcal{C}, r \in \mathcal{R} \tag{2}$$

B. Cost Model of RSU

The storage space allocated by the RSUs is utilized by the CPs to cache the content during the placement phase. Thus, the RSU consumes a certain amount of energy for receiving contents from CPs [17], [18]. We model the energy cost of RSU r using a bi-dimensional function $\mathcal{V}_{rc}(y_{rc}, \phi_{rc})$ for receiving content from the c^{th} CP where y_{rc} signifies the amount of storage space allocated by RSU r to the c^{th} CP. We assume that the cost $\mathcal{V}_{rc}(\phi_{rc}, y_{rc})$ is strictly convex, positive, and increasing function of y_{rc}. The cost function captures the fact that the cost of RSU r not only depends upon the total allocated amount of storage space $Y_r = \sum_{c \in \mathcal{C}} y_{rc}$ but also depends on the storage allocation vector of the RSU ($\boldsymbol{y_r} = (y_{r1}, y_{r2}, \dots y_{rC})$), i.e., the amount of storage space allocated to individual CPs. Further, the RSU may incur different costs for offering storage space to different CPs due to the asymmetric backhaul link capacity between RSU and CPs. For example, the RSU consumes more energy for receiving a certain amount of content when the connecting backhaul link capacity is low. We capture this fact by defining a reluctance parameter $\phi_{rc} \in (0, 1]$ of RSU $r \in \mathcal{R}$ for CP $c \in \mathcal{C}$ where $\phi_{rc} = 1$ signifies that RSU r is not interested to cache the content of CP c. Since it is beneficial for the RSU to allocate storage to the CP for whom the reluctance is low, the cost $\mathcal{V}_{rc}(\cdot)$ is an increasing function of ϕ_{rc}. Specifically, we define the cost of RSU for all the CPs as follows:

$$\mathcal{V}_r(\boldsymbol{y_r}, \boldsymbol{\phi}_r) = \sum_{c \in \mathcal{C}} \mathcal{V}_{rc}(\phi_{rc}, y_{rc}) \tag{3}$$

where $\boldsymbol{\phi}_r = (\phi_{rc})_{c \in \mathcal{C}}$.

Further, RSU r is constrained by the total available storage capacity S_r, the total storage space provisioned by RSU should satisfy the following constraint:

$$\sum_{c \in \mathcal{C}} y_{rc} \leq S_r \qquad \forall r \in \mathcal{R} \tag{4}$$

The objective of RSU is to minimize its cost ($\mathcal{V}$) by allocating less amount storage space to CPs whereas the CP tries to maximize its utility ($\mathcal{U}$) by requesting more amount of storage space from RSUs. Since the objectives of the CPs and RSUs are conflicting in nature it is difficult to reach on mutual agreement, if CPs and RSUs decide independently their concerned variables, i.e., $\boldsymbol{x}$ and $\boldsymbol{y}$. Thus, we need a market maker (central entity) which determines the allocation of RSUs' storage among requesting CPs to maximize the social welfare (SW).

IV. Problem Formulation

In the considered RSU-based caching system, the set of CPs compete for the limited storage space of the RSUs. A key issue in our storage allocation problem is to define the notion of efficiency which focuses not only on the utilization of available storage space of the RSUs, but also on the optimal allocation of the storage space of the RSUs among the competing CPs. Further, the utility function of the CPs and the cost function of the RSUs, corresponding to a storage allocation, are conflicting in nature. Thus, it is difficult for the RSUs and CPs to reach a mutually beneficial agreement. Our proposed scheme incorporates the utility and cost functions of the participating entities and aims to maximize the *societal welfare* by allocating the storage space of the RSUs to those CPs who value them the most (i.e., Pareto optimally). Thus, motivated by economics theory [19], [20], we consider social welfare, denoted by $\Psi(\boldsymbol{x}, \boldsymbol{y})$, as an important parameter which is defined as the aggregated utility of the CPs and total cost of the RSUs. Mathematically,

$$\begin{aligned}\Psi(\boldsymbol{x}, \boldsymbol{y}) &= \sum_{c\in\mathcal{C}} \mathcal{U}_c(\cdot) + \sum_{r\in\mathcal{R}} -(\mathcal{V}_r(\cdot)) \\ &= \sum_{c\in\mathcal{C}} \mathcal{U}_c(\cdot) - \sum_{r\in\mathcal{R}} \mathcal{V}_r(\cdot)\end{aligned} \tag{5}$$

The MM also consider the individual constraints of the CPs and the RSUs to maximize the social welfare. Specifically, MM solves the following optimization problem.

$$P1: \max_{(\boldsymbol{x}\geq 0, \boldsymbol{y}\geq 0)} \Psi(\boldsymbol{x}, \boldsymbol{y}) \tag{6}$$

$$\text{s.t.} \quad (2), (4)$$

$$x_{cr} = y_{rc} \qquad \forall c \in \mathcal{C}, r \in \mathcal{R} \tag{7}$$

where the first and second constraints are the backhaul and capacity constraints, respectively. Further, the third constraint is the market equilibrium condition which ensures that the CP-RSU pairs agree on the allocated amount of storage space. Finally, the last constraint is the feasibility constraint. The given maximization problem $P1$ is convex in nature since the objective function is concave and the set of constraints construct a convex and compact feasible region. Thus, there exists a unique optimal solution $(\boldsymbol{x}^\dagger, \boldsymbol{y}^\dagger)$. The Lagrange function corresponding to the $P1$ is given in Eqn. (8), where, $\boldsymbol{\pi}^{C\times R}$, $\boldsymbol{\lambda}^{1\times R}$, and $\boldsymbol{\mu}^{C\times R}$ are the Lagrange multipliers corresponding to the backhaul, capacity, and market equilibrium constraints, respectively.

Next, we use the necessary and sufficient Karush-Kuhn-Tucker (KKT) conditions [21] to find the set of equations given in ($\boldsymbol{E_{11}}$-$\boldsymbol{E_{17}}$). The Eqns. $\boldsymbol{E_{11}}$ and $\boldsymbol{E_{12}}$ are the Stationarity conditions and Eqns. $\boldsymbol{E_{13}}$-$\boldsymbol{E_{15}}$ are the Complementary Slackness conditions. Finally, the Primal and Dual Feasibility conditions are given in Eqns. $\boldsymbol{E_{16}}$-$\boldsymbol{E_{18}}$. The optimal solution of $P1$, i.e., $\boldsymbol{x}^\dagger, \boldsymbol{y}^\dagger, \boldsymbol{\lambda}^\dagger, \boldsymbol{\mu}^\dagger$, and $\boldsymbol{\pi}^\dagger$, can be obtained by solving the set of equations ($\boldsymbol{E_{11}}$ - $\boldsymbol{E_{17}}$) simultaneously.

$$\boldsymbol{E_{11}}: \frac{\partial \mathcal{U}_c(\boldsymbol{x}_c^\dagger)}{\partial x_{cr}} = \mu_{cr}^\dagger + \pi_{cr}^\dagger \qquad \boldsymbol{E_{12}}: \frac{\partial \mathcal{V}_r(\boldsymbol{y}_r^\dagger)}{\partial y_{rc}} = \mu_{cr}^\dagger - \lambda_r^\dagger$$

$$\boldsymbol{E_{13}}: \lambda_r^\dagger\left(\sum_{c\in\mathcal{C}} y_{rc}^\dagger - S_r\right) = 0 \qquad \boldsymbol{E_{14}}: \pi_{cr}^\dagger(x_{cr}^\dagger - b_{cr}^\dagger) = 0$$

$$\boldsymbol{E_{15}}: \mu_{cr}^\dagger.(y_{rc}^\dagger - x_{cr}^\dagger) = 0 \qquad \boldsymbol{E_{16}}: x_{cr}^\dagger = y_{rc}^\dagger$$

$$\boldsymbol{E_{17}}: x_{cr}^\dagger, y_{rc}^\dagger, \lambda_r^\dagger \geq 0, \pi_{cr}^\dagger \geq 0$$

However, it is not possible for the MM to solve $P1$ because the MM does not have complete information about the CPs and RSUs. In particular, the utility function of CPs and cost function of RSUs are their private information and not known to the MM. Additionally, as both the CP and the RSU are strategic they may reveal their information untruthfully when asked by the MM resulting market manipulation. Therefore, for truthful extraction of RSUs' and CPs' private information, there is need for designing an incentive mechanism for MM. The double auction is a widely used mechanism to handle the information asymmetry among multiple buyers and sellers [14], [22]. In our case, the MM acts as an auctioneer and initiates auction for the available storage space wherein the CPs act as a buyer and RSUs act as a seller. The CP pays to the MM for the allocated storage space and the MM transfers the payment to the RSUs to compensate for their cost.

V. Mechanism Design

In this section, we discuss the double auction mechanism which motivates the strategic agents (CPs and RSUs) to trade storage space and also satisfy the following four economic properties:

- Efficient: The outcome of the mechanism, i.e., the storage allocation among CPs should maximize the social welfare defined in Eqn. (5).
- Incentive compatible: The mechanism should able to elicit the private information of the participating CPs and RSUs.
- Individually rational: The mechanism should ensure non-negative payoff for the participating CPs and RSUs.
- Budget balance: The budget of the MM is the difference between the payment received from the CPs and the reimbursement paid to the RSUs. The proposed mechanism should guarantee that the total reimbursement to the RSUs

$$\mathcal{L}_1(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{\lambda}, \boldsymbol{\mu}, \boldsymbol{\pi}) = \Psi(\cdot) - \sum_{r\in\mathcal{R}} \lambda_r\left(\sum_{c\in\mathcal{C}} y_{rc} - S_r\right) - \sum_{c\in\mathcal{C}}\sum_{r\in\mathcal{R}} \pi_{cr}(x_{cr} - b_{cr}) - \sum_{c\in\mathcal{C}}\sum_{r\in\mathcal{R}} \mu_{cr}(x_{cr} - y_{rc}) \tag{8}$$

should not exceed the aggregated payment received from the CPs, i.e., the MM should not bear any monetary loss.

However, there is no such double auction mechanism which follows aforementioned four economic properties simultaneously [15], [23]–[25]. Therefore, we present an iteration-based double-sided auction (IDA) mechanism. The core idea of IDA mechanism is that the MM solves an alternative optimization problem (as MM is not aware of utility and cost functions) to compute RSUs' storage space allocation to CPs. The allocation is combined with payment (for the CPs) and reimbursement (for the RSUs) scheme in order to maximize social welfare (Eqn. (5)).

A. MM's Optimization Problem

Let CP c submits bid vector $\boldsymbol{\beta}_c = (\beta_{c1}, \beta_{c2}, \ldots \beta_{cR})$ to the MM where β_{cr} signifies the amount of storage space CP c needs from RSU r. Likewise, RSU $r \in \mathcal{R}$ submits its bid vector $\boldsymbol{\alpha}_r = (\alpha_{r1}, \alpha_{r2}, \ldots \alpha_{rC})$ to the MM where α_{rc} signifies the amount of storage space RSU r allocates to CP c. Based on bids of CPs and RSUs, the MM's goal is to find the storage allocation that each RSU assigns to CPs by solving the optimization problem $P2$.

$$P2: \max_{(\boldsymbol{x}\geq 0, \boldsymbol{y}\geq 0)} \sum_{r\in\mathcal{R}} \sum_{c\in\mathcal{C}} \left(\beta_{cr} \log x_{cr} - \frac{\alpha_{rc} y_{rc}^2}{2} \right) \tag{9}$$
$$\text{s.t.} \quad (2), (4), (7)$$

The objective function of problem $P2$ is derived from [13], [14], [22]. The first part of the objective function captures the increasing utility of CPs and its second part models the increasing cost of RSUs for storage allocation. Intuitively, the MM aims to maximize the pairwise gain of CPs and RSUs for storage allocation. Note that the constraints of the problem $P2$ are similar to that of the problem $P1$ and construct a convex and compact feasible region. Further, the objective function is concave. Thus, problem $P2$ is a convex optimization problem and has a unique optimal solution $(\boldsymbol{x}^\ddagger, \boldsymbol{y}^\ddagger)$. The Lagrangian function $\mathcal{L}_2(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{\lambda}, \boldsymbol{\mu}, \boldsymbol{\pi})$ corresponding to $P2$ is given in Eqn. (10).

The optimal solution – $\boldsymbol{x}^\ddagger, \boldsymbol{y}^\ddagger, \boldsymbol{\lambda}^\ddagger, \boldsymbol{\mu}^\ddagger, \boldsymbol{\pi}^\ddagger$, can be obtained by solving the following system of equations simultaneously. The five KKT conditions of $P2$ (i.e., $\boldsymbol{E_{23}} - \boldsymbol{E_{27}}$) are similar

$\boldsymbol{E_{21}}: x^\ddagger_{cr} = \frac{\alpha_{rc}}{\mu^\ddagger_{cr} + \pi^\ddagger_{cr}}$ $\quad$ $\boldsymbol{E_{22}}: y^\ddagger_r = \frac{\mu^\ddagger_{cr} - \lambda^\ddagger_r}{\alpha_{rc}}$

$\boldsymbol{E_{23}}: \lambda^\ddagger_r \left(\sum_{c\in\mathcal{C}} y^\ddagger_{rc} - S_r \right) = 0$ $\quad$ $\boldsymbol{E_{24}}: \pi^\ddagger_{cr}(x^\ddagger_{cr} - b^\ddagger_{cr}) = 0$

$\boldsymbol{E_{25}}: \mu^\ddagger_{cr}.(y^\ddagger_{rc} - x^\ddagger_{cr}) = 0$ $\quad$ $\boldsymbol{E_{26}}: x^\ddagger_{cr} = y^\ddagger_{rc}$

$\boldsymbol{E_{27}}: x^\ddagger_{cr}, y^\ddagger_{rc}, \lambda^\ddagger_r, \pi^\ddagger_{cr} \geq 0$

to that of the $P1$ (i.e., $\boldsymbol{E_{13}} - \boldsymbol{E_{17}}$). However, the first two KKT conditions are different. By comparing KKT conditions $\boldsymbol{E_{11}}$ and $\boldsymbol{E_{12}}$ with $\boldsymbol{E_{21}}$ and $\boldsymbol{E_{22}}$, we find that if CPs and RSUs bid according to Eqn. (11) and (12), respectively, then the optimal solution of $P2$ also maximizes the social welfare defined in eq. (6).

$$\beta_{cr} = x^\ddagger_{cr} . \frac{\partial \mathcal{U}_c(\boldsymbol{x}^\ddagger)}{\partial x_{cr}} \tag{11}$$

$$\alpha_{rc} = \frac{1}{y^\ddagger_{rc}} . \frac{\partial \mathcal{V}_r(\boldsymbol{y}^\ddagger)}{\partial y_{rc}} \tag{12}$$

The ultimate goal of the MM is to design payment and reimbursement rules for CPs and RSUs, respectively, which motivate them to attain the desired bid defined in Eqns. (11) and (12). Let $\mathcal{P}_c(\boldsymbol{\beta}_c)$ denote the payment paid by the CP c to the MM, in correspondence to bid $\boldsymbol{\beta}_c$. Likewise, $\mathcal{Q}_r(\boldsymbol{\alpha}_r)$ denotes the reimbursement received by the RSU r, from the MM, in correspondence to bid $\boldsymbol{\alpha}_r$.

CP Optimization Problem: Based on the allocation rule $\boldsymbol{E_{21}}$ and the payment rule $\mathcal{P}_c(\boldsymbol{\beta}_c)$ CP c solves the following optimization problem to find the best response, i.e., $\boldsymbol{\beta}_c$.

$$CP-OP: \quad \max_{\boldsymbol{\beta}_c} \quad \Big(\mathcal{U}_c(\boldsymbol{x}_c) - \mathcal{P}_c(\boldsymbol{\beta}_c) \Big) \tag{13}$$
$$\text{s.t.} \quad \beta_{cr} \geq 0, \quad \forall r \in \mathcal{R} \tag{14}$$

The $CP-OP$ problem yields the optimal value at the given condition.

$$\frac{\partial \mathcal{P}_c(\boldsymbol{\beta}_c)}{\partial \beta_{cr}} = \frac{1}{\mu_{cr}} . \frac{\partial \mathcal{U}_c(\boldsymbol{x}_c)}{\partial x_{cr}} \tag{15}$$

RSU optimization problem: Based on the allocation and the reimbursement rule $\boldsymbol{E_{22}}$ and $\mathcal{Q}_r(\boldsymbol{\alpha}_r)$, respectively, RSU r solves the following optimization problem to find its best response, i.e., $\boldsymbol{\alpha}_r$.

$$RSU-OP: \quad \max_{\boldsymbol{\alpha}_r} \quad \Big(\mathcal{Q}_r(\boldsymbol{\alpha}_r) - \mathcal{V}_r(\boldsymbol{y}_r) \Big) \tag{16}$$
$$\text{s.t.} \quad \alpha_{rc} \geq 0, \quad \forall c \in \mathcal{C} \tag{17}$$

The $RSU-OP$ problem yields the optimal value at the given condition.

$$\frac{\partial \mathcal{Q}_r(\boldsymbol{\alpha}_r)}{\partial \alpha_{rc}} = \frac{\lambda_r - \mu_{cr}}{\alpha_{rc}^2} . \frac{\partial \mathcal{V}_r(\boldsymbol{y}_r)}{\partial y_{rc}} \tag{18}$$

B. Payment and Reimbursement Function

The MM finds the payment and the reimbursement functions based on the best response of CPs and RSUs. Comparing the desirable bid of the CP in Eqn. (11) with the its best response defined in Eqn. (15) we obtain

$$\frac{\partial \mathcal{P}_c(\boldsymbol{\beta}_c)}{\partial \beta_{cr}} = \frac{1}{\mu_{cr}} . \frac{\beta_{cr}}{x_{cr}} \tag{19}$$

Further, replacing x_{cr} from allocation rule $\boldsymbol{E_{21}}$, we rewrite the above equation

$$\frac{\partial \mathcal{P}_c(\boldsymbol{\beta}_c)}{\partial \beta_{cr}} = 1 \tag{20}$$

$$\mathcal{L}_2(\cdot) = \sum_{c\in\mathcal{C}} \sum_{r\in\mathcal{R}} \left(\beta_{cr} \log x_{cr} - \frac{\alpha_{rc} y_{rc}^2}{2} \right) - \sum_{r\in\mathcal{R}} \lambda_r \left(\sum_{c\in\mathcal{C}} y_{rc} - S_r \right) - \sum_{c\in\mathcal{C}} \sum_{r\in\mathcal{R}} \pi_{cr}(x_{cr} - b_{cr}) - \sum_{c\in\mathcal{C}} \sum_{r\in\mathcal{R}} \mu_{cr}(x_{cr} - y_{rc}) \tag{10}$$

Hence, the payment function for CP c is as follows:

$$\mathcal{P}_c(\boldsymbol{\beta}_c) = \sum_{r\in\mathcal{R}} \beta_{cr} \tag{21}$$

Similarly, using the allocation rule $\boldsymbol{E_{22}}$ and the best response of RSU defined in Eqn. (18), the MM decides the reimbursement rule $\mathcal{Q}_r$ for RSU r as follows:

$$\mathcal{Q}_r(\boldsymbol{\alpha}_r) = \sum_{c\in\mathcal{C}} \frac{(\mu_{cr}-\lambda_r)^2}{\alpha_{rc}} = \sum_{c\in\mathcal{C}} y_{rc}(\mu_{cr}-\lambda_r) \tag{22}$$

In summary, the MM allocates the storage space to the CP according to its need (see eq. $\boldsymbol{E_{21}}$) and reimburses RSUs proportional to the amount of their storage space. This allocation and pricing scheme maximizes the social welfare defined in Eqn. (5).

C. Economic Property Analysis

In this section, we systematically present the analytical proof of the economic properties.

Proposition 1. *The proposed IDA mechanism maximizes the social welfare function defined in Eqn. 5 and is hence efficient.*

Proof: For the efficiency of the proposed IDA mechanism, it is sufficient to show that the Algorithm 1 converges at a social welfare point, i.e., satisfies Eqns. $\boldsymbol{E_{11}}$ - $\boldsymbol{E_{17}}$.

At convergence, the bidding strategy of CP Eqn. (15) and the pricing rule Eqn. (21) imply $\frac{\partial \mathcal{U}_c(\boldsymbol{x}_c^{\ddagger})}{\partial x_{cr}} = \mu_{cr}^{\ddagger}$. Similarly, the RSU's strategy Eqn. (18) and the reimbursement rule Eqn. (22) imply $\frac{\partial \mathcal{V}_r(\boldsymbol{y}_r^{\ddagger})}{\partial y_{bh}} = \mu_{cr}^{\ddagger} - \lambda_r^{\ddagger}$. These implications, along with the preposition 5, show that, at convergence, the KKT conditions ($\boldsymbol{E_{11}}$ - $\boldsymbol{E_{17}}$) are satisfied. Hence, the IDA mechanism is efficient. ■

Proposition 2. *The proposed IDA mechanism motivates CPs and RSUs to bid truthfully, and is hence incentive compatible or strategy proof.*

Proof: The optimization problem of CPs and RSUs, in Eqn. (13) and Eqn. (16), are comprised of the pricing and the reimbursement rules, which motivates them to update their bids gradually and attain the social welfare bid, as defined in Eqn. (11) and Eqn. (12). Additionally, the proposed IDA mechanism allows CPs and RSUs to solve their individual optimization problem in each iteration. For the CP and the RSU submitting the true bid value is the best strategy and hence, IDA mechanism is incentive compatible. ■

Proposition 3. *The proposed IDA mechanism is individually rational, i.e., at convergence, CPs and RSUs have non-negative payoff.*

Proof: For the individual rationality of the proposed system, first we show that each CP ($c \in \mathcal{C}$) has non-negative payoff, which is expressed as,

$$\mathcal{U}_c(\boldsymbol{x}_c^{\ddagger}) - \sum_{r=1}^{R} \beta_{cr} \geq 0 \tag{23}$$

Substituting Eqn. (11) in Eqn. (23), we get

$$\mathcal{U}_c(\boldsymbol{x}_c^{\ddagger}) \geq \sum_{r=1}^{R} x_{cr}^{\ddagger} \frac{\partial \mathcal{U}_c(\boldsymbol{x}_c^{\ddagger})}{\partial x_{cr}} \tag{24}$$

As the utility function ($\mathcal{U}_c(\boldsymbol{x}_c^{\ddagger})$) of CP ($c \in \mathcal{C}$) is a concave function, we have,

$$\mathcal{U}_c(0) - \mathcal{U}_c(\boldsymbol{x}_c^{\ddagger}) \leq \nabla \mathcal{U}_c(\boldsymbol{x}_c^{\ddagger})^T (0 - \boldsymbol{x}_c^{\ddagger}) \tag{25}$$

Since, $\mathcal{U}_c(0) = 0$, the above equation is similar to the inequality derived in Eqn. (24).

Similarly, for each RSU ($r \in \mathcal{R}$), we get the following condition

$$\mathcal{V}_r(\boldsymbol{y}_{rc}^{\ddagger}) \geq \sum_{b=1}^{B} y_{rc}^{\ddagger} \frac{\partial \mathcal{V}_r(\boldsymbol{y}_{rc}^{\ddagger})}{\partial y_{rc}} \tag{26}$$

As the cost function ($\mathcal{V}_r(\boldsymbol{y}_{rc}^{\ddagger})$) of RSU ($r \in \mathcal{R}$) is a convex function, we have,

$$\mathcal{V}_r(0) - \mathcal{V}_r(\boldsymbol{y}_{rc}^{\ddagger}) \geq \nabla \mathcal{V}_r(\boldsymbol{y}_{rc}^{\ddagger})^T (0 - \boldsymbol{y}_{rc}^{\ddagger}) \tag{27}$$

Since, $\mathcal{V}_r(0) = 0$, the above equation is similar to the inequality derived in Eqn. (26). ■

Proposition 4. *The proposed IDA mechanism is budget balanced (weakly), i.e., the MM does not need to invest money for proper functioning of the market.*

Proof: Let $\mathbf{BB}(\boldsymbol{\beta}, \boldsymbol{\alpha})$ be the budget of the MM.

$$\begin{aligned}\mathbf{BB}(\boldsymbol{\beta}, \boldsymbol{\alpha}) &= \sum_{c\in\mathcal{C}} \mathcal{P}_c(\boldsymbol{\beta}_c) - \sum_{r\in\mathcal{R}} \mathcal{Q}_r(\boldsymbol{\alpha}_r) \\ &= \sum_{c=1}^{C}\sum_{r=1}^{R} \beta_{cr} - \sum_{c=1}^{C}\sum_{r=1}^{R} \frac{(\lambda_r - \mu_{cr})^2}{\alpha_{rc}}\end{aligned} \tag{28}$$

Substituting Eqns. $\boldsymbol{E_{21}}$ and $\boldsymbol{E_{22}}$ in Eqn. (28), we get,

$$\begin{aligned}\mathbf{BB}(\boldsymbol{\beta}, \boldsymbol{\alpha}) &= \sum_{c=1}^{C}\sum_{r=1}^{R} x_{cr}^{\ddagger}\mu_{cr}^{\ddagger} + \sum_{c=1}^{C}\sum_{r=1}^{R} y_{rc}^{\ddagger}(\lambda_r^{\ddagger} - \mu_{cr}^{\ddagger}) \\ &= \sum_{c=1}^{C}\sum_{r=1}^{R} \mu_{cr}^{\ddagger}(x_{cr}^{\ddagger} - y_{rc}^{\ddagger}) + \sum_{c=1}^{C}\sum_{r=1}^{R} y_{rc}^{\ddagger}\lambda_r^{\ddagger} \\ &\geq 0\end{aligned} \tag{29}$$

The last inequality is true because at the equilibrium, $x_{cr}^{\ddagger} = y_{rc}^{\ddagger}$ diminishes the first term. Further, $\lambda_r^{\ddagger} \geq 0$ due to the complementary slackness condition and $y_{rc}^{\ddagger} \geq 0$. ■

In the next section, we discuss the algorithm which enables the implementation of proposed IDA mechanism.

VI. IMPLEMENTATION OF IDA MECHANISM

The MM is not aware of the private information (utility and cost functions) of the participating CPs and RSUs. Hence, to reach the market equilibrium condition, i.e., ($\boldsymbol{x}^{\ddagger} = \boldsymbol{y}^{\ddagger}$), for each CP-RSU pair, we proposed a two-phase iterative algorithm. In the first phase, the CPs and RSUs submit their bids to the MM according to their best responses which is obtained by solving their individual optimization problems (defined in Eqns.(15) and (18)). Specifically, the submitted bids *signal* the preference and reluctance of the CPs and RSUs. In the second phase, the MM computes the storage allocation according to

Eqns. E_{21} and E_{22}, and the payment and reimbursement of the CPs and RSUs based on the submitted bids, without knowing their utility and cost functions. The algorithm converges when there is an insignificant change in the bids of the CPs and RSUs.

Algorithm 1: Two Phase Iterative Algorithm

Inputs : δ, ϵ
Outputs: $\boldsymbol{x}^{\ddagger}$, $\boldsymbol{y}^{\ddagger}$, $\mathcal{P}$, $\mathcal{Q}$
Initialize and announce μ_{cr}^{0}, λ_r^0 $\forall c \in \mathcal{C}, \forall r \in \mathcal{R}$ to corresponding CP and RSU
$converge = 0, t = 0$
while *converge = 0* **do**
  $t \leftarrow t+1$
  Phase-I
  1) Each CP computes the optimal bid $\boldsymbol{\beta}_c^t$ using eq. (15).
  2) Each RSU computes the optimal bid $\boldsymbol{\alpha}_r^t$ using eq. (18).
  Phase-II
  1) The MM computes the cache allocation and acquisition ($\boldsymbol{x}^t$ and $\boldsymbol{y}^t$), using eq. $\boldsymbol{E_{21}}$ and $\boldsymbol{E_{22}}$.
  2) The MM computes the payment and reimbursement ($\mathcal{P}^t$, $\mathcal{Q}^t$), using eq. (21) and (22).
  3) The MM update the dual variables ($\boldsymbol{\lambda}^{t+1}$, $\boldsymbol{\mu}^{t+1}$, $\boldsymbol{\pi}^{t+1}$) using eq. (31) - (33).
  4) **if** *change in CPs and RSUs bids are* $\leq \epsilon$ **then**
    converge = 1

A. Algorithm

The algorithm functions in two phases. In the first phase, each CP and RSU compute its optimal bid using Eqn. (15) and (18), respectively. In the second phase, the MM employs a primal-dual Lagrange decomposition approach to solve the problem $P2$ due to its decomposable structure. The MM computes the $\boldsymbol{x}^t$ and $\boldsymbol{y}^t$ followed by the $\mathcal{P}_c^t$ and $\mathcal{Q}_r^t$ in each iteration t. The MM updates the dual variables using the sub-gradient decent methods, as follows:

$$\lambda_r^{t+1} = \left(\lambda_r^t - \delta \frac{\partial \mathcal{L}_2(.)}{\partial \lambda_r}\right)^+ \quad \forall r \in \mathcal{R} \tag{30}$$

$$\mu_{cr}^{t+1} = \left(\mu_{cr}^t - \delta \frac{\partial \mathcal{L}_2(.)}{\partial \mu_{cr}}\right) \quad \forall c \in \mathcal{C}, \forall r \in \mathcal{R} \tag{31}$$

$$\pi_{cr}^{t+1} = \left(\pi_{cr}^t - \delta \frac{\partial \mathcal{L}_2(.)}{\partial \pi_{cr}}\right) \quad \forall c \in \mathcal{C}, \forall r \in \mathcal{R} \tag{32}$$

where $\delta > 0$ is the step size $(.)^+$, which ensures that the Lagrange multipliers corresponding to the constraint given in Eqn. (4) attains non-negative values, i.e., $\lambda_r^{t+1} \geq 0$. The iteration continues till the difference between the bids submitted by the CP and the RSU in two consecutive iterations are sufficiently small.

B. Tolerance value

In Algorithm 1, the tolerance value ϵ signifies the difference between the optimal social welfare (SW) value and the SW value at the convergence, obtained through the proposed iterative double auction mechanism. In particular, ϵ denotes the acceptable error between the optimal and converged SW values. Thus, the larger the value of ϵ, the smaller the number of iterations required for convergence of the algorithm. Next, we present the convergence proof of the proposed algorithm.

Proposition 5. *The proposed two-phase iterative algorithm converges to the solution point of the problem $P2$, starting from any point which fulfills the complementary slackness conditions $\boldsymbol{E_{23}} - \boldsymbol{E_{27}}$.*

Proof: To make the analysis tractable we assume that the step-size (δ) is very small, i.e., the time required to update the dual variables is very small. Thus, the analysis can be done in the continuous-time domain. Therefore, from Eqns. (31), (32) and (33), the rate of update of Lagrange multiplier is

$$\frac{d\lambda_r}{d\delta} = \left(\sum_{c \in \mathcal{C}} y_{rc} - S_r\right)^+_{\lambda_r}, \forall r \in \mathcal{R} \tag{33}$$

$$\frac{d\mu_{cr}}{d\delta} = (y_{rc} - x_{cr})_{\mu_{cr}} \quad \forall c \in \mathcal{C}, \forall r \in \mathcal{R} \tag{34}$$

$$\frac{d\pi_{cr}}{d\delta} = (x_{cr} - b_{cr})^+_{\pi_{cr}} \quad \forall c \in \mathcal{C}, \forall r \in \mathcal{R} \tag{35}$$

where, the notation $(.)^+$ denotes the projection on nonnegative orthant. Further, for the proof on convergence we define the Lyapunov function as

$$L(\boldsymbol{\lambda}, \boldsymbol{\mu}, \boldsymbol{\pi}) = \sum_{r=1}^{R} \frac{(\lambda_r - \lambda_r^{\ddagger})^2}{2} + \sum_{c=1}^{C}\sum_{r=1}^{R} \frac{(\mu_{cr} - \mu_{cr}^{\ddagger})^2}{2} + \sum_{c=1}^{C}\sum_{r=1}^{R} \frac{(\pi_{cr} - \pi_{cr}^{\ddagger})^2}{2} \tag{36}$$

Our goal is to prove that $\frac{d\mathcal{L}_2(\boldsymbol{\lambda},\boldsymbol{\mu},\boldsymbol{\pi})}{d\delta} \leq 0$. Taking derivatives on both sides of Eqn. (37) with respect to t and substituting Eqns. (34), (35) and (36), we obtain $\frac{dL(\cdot)}{d\delta}$ as given in Eq. (38)

The reason behind the inequality is, when $\lambda_r > 0$, we have $(.)^+_{\lambda_r} = (.)$ and $(\lambda_r - \lambda_r^{\ddagger})(.)^+_{\lambda_r} = (\lambda_r - \lambda_r^{\ddagger})(.)$. On the other hand, when $\lambda_r = 0$, we have $(.)^+_{\lambda_r} \geq (.)$ and $(0 - \lambda_r^{\ddagger})(.)^+_{\lambda_r} \geq (0 - \lambda_r^{\ddagger})(.)$. Hence, for any value of λ_r, we have $(\lambda_r - \lambda_r^{\ddagger})(.)^+_{\lambda_r} \leq (\lambda_r - \lambda_r^{\ddagger})(.)$. After evaluating $\sum_{c=1}^{C} y_{rc}^{\ddagger}$, $(x_{cr}^{\ddagger} - y_{rc}^{\ddagger})$, and $(x_{cr}^{\ddagger} - b_{cr})$ in the RHS of the above inequality

$$\begin{aligned} \frac{dL(\cdot)}{d\delta} &= \sum_{r=1}^{R}((\lambda_r - \lambda_r^{\ddagger}))\left(\sum_{c \in \mathcal{C}} y_{rc} - S_r\right)^+_{\lambda_r} + \sum_{c=1}^{C}\sum_{r=1}^{R}(\mu_{cr} - \mu_{cr}^{\ddagger})(y_{rc} - x_{cr})_{\mu_{cr}} + \sum_{c=1}^{C}\sum_{r=1}^{R}(\pi_{cr} - \pi_{cr}^{\ddagger})(x_{cr} - b_{cr})^+_{\pi_{cr}} \\ &\leq \sum_{r=1}^{R}((\lambda_r - \lambda_r^{\ddagger}))\left(\sum_{c \in \mathcal{C}} y_{rc} - S_r\right) + \sum_{c=1}^{C}\sum_{r=1}^{R}(\mu_{cr} - \mu_{cr}^{\ddagger})(y_{rc} - x_{cr}) + \sum_{c=1}^{C}\sum_{r=1}^{R}(\pi_{cr} - \pi_{cr}^{\ddagger})(x_{cr} - b_{cr})_{\pi_{cr}} \end{aligned} \tag{37}$$

and using complementary slackness conditions along with equations E_{21} and E_{22}, we get

$$\frac{dL(\cdot)}{d\delta} = \sum_{c=1}^{C}\sum_{r=1}^{R}(x_{cr} - x^{\ddagger}_{cr})\left(\frac{\partial \mathcal{U}_c(\boldsymbol{x}_c)}{\partial x_{cr}} - \frac{\partial \mathcal{U}_c(\boldsymbol{x}^{\ddagger}_c)}{\partial x_{cr}}\right) + \sum_{c=1}^{C}\sum_{r=1}^{R}(y_{rc} - y^{\ddagger}_{rc})\left(\frac{\partial \mathcal{V}_r(\boldsymbol{y}^{\ddagger}_{rc})}{\partial \psi_h} - \frac{\partial \mathcal{V}_r(\boldsymbol{y}_r)}{\partial y_{rc}}\right) \quad (38)$$

Since the utility function ($\mathcal{U}_c$) of CP ($c \in \mathcal{C}$) is strictly concave and the utility function ($\mathcal{V}_r$) of RSU ($r \in \mathcal{R}$) is strictly convex in nature, we have,

$$\left(\frac{\partial \mathcal{U}_c(\boldsymbol{x}_c)}{\partial x_{cr}} - \frac{\partial \mathcal{U}_c(\boldsymbol{x}^{\ddagger}_c)}{\partial x_{cr}}\right)(x_{cr} - x^{\ddagger}_{cr}) \leq 0 \quad (39)$$

$$\left(\frac{\partial \mathcal{V}_r(\boldsymbol{y}^{\ddagger}_{rc})}{\partial \psi_h} - \frac{\partial \mathcal{V}_r(\boldsymbol{y}_r)}{\partial y_{rc}}\right)(y_{rc} - y^{\ddagger}_{rc}) \leq 0 \quad (40)$$

Therefore, we conclude that $\frac{dL(\cdot)}{d\delta} \leq 0$, and the given algorithm converges. ■

VII. Simulation Results

In this section, we provide simulation results to show the effectiveness and efficacy of the proposed IDA mechanism. We also present the effect of various system parameters on the social welfare of the RSU-based caching system. All the numerical simulations are performed using MATLAB. We consider a small RSU-based caching system with $R = 6$ RSUs as sellers and $C = 3$ CPs as buyers. Each RSU is enabled with finite storage capacity $S_r = 16$ GB. Each CP is connected to RSU with a backhaul link of capacity $b_{cr} = 10$ GB. The preference of CPs towards all RSUs (i.e. θ_{cr}) is shown in Table I. Similarly, Table II presents the service cost parameter of RSUs for each CPs (i.e. ϕ_{rc}). We choose $\mathcal{U}_c = 10.\sum_{r\in\mathcal{R}}(\theta_{cr}x_{cr})^{0.3}$ and $\mathcal{V}_r = 0.1.\sum_{r\in\mathcal{R}} e^{(\phi_{rc}y_{rc})}$ to model the utility and cost of c^{th} CP and r^{th} RSU, respectively.

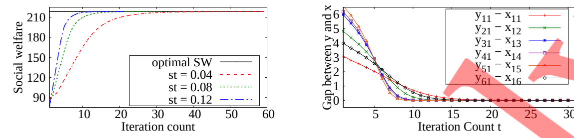
TABLE I: Preference parameters of CPs

	RSU1	RSU2	RSU3	RSU4	RSU5	RSU6
CP1	0.95	0.71	0.72	0.68	0.64	0.58
CP2	0.95	0.75	0.61	0.73	0.80	0.61
CP3	0.68	0.64	0.51	0.90	0.87	0.73

TABLE II: Serving cost parameter of RSUs

	RSU1	RSU2	RSU3	RSU4	RSU5	RSU6
CP1	0.88	0.67	0.58	0.57	0.54	0.77
CP2	0.53	0.92	0.62	0.90	0.60	0.93
CP3	0.56	0.99	0.50	0.82	0.65	0.80

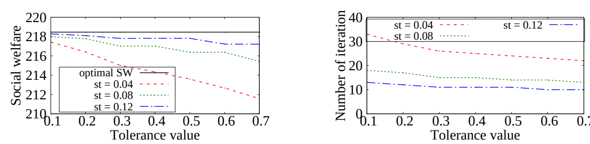
Convergence analysis: In Fig. 2(a), we depict the evolution of social welfare obtained by the proposed iterative algorithm in each iteration and its convergence to maximum achievable social welfare value (MSW). In this case, MSW is obtained (black solid line in Fig. 2(a)) by solving the problem $P1$ centrally when assuming all the required information (such as utility, cost function, storage capacity, and maximum demand) are known to the MM. Further, the effect of different step-size on the convergence of the proposed algorithm to the MSW is shown in Fig. 2(a). We observe that with an increase in step-size the rate of convergence of proposed algorithm increases. Specifically, when step-size is 0.04 the proposed algorithm converges after 59 iterations, whereas for step-size 0.12 the algorithm requires only 21 iterations. This verifies the convergence of the proposed algorithm as proved in proposition 5.



(a) Effect of step-size on the evolution of SW (b) Evoluation of gap for CP1

Fig. 2: Convergence analysis

The evolution of gap between storage allocation (y) and storage demand (x) in each iteration for CP1 is shown in Fig. 2(b). In particular, we show the difference between the storage allocation and demand of CP1 from different RSUs. We observe that the difference between the allocation and demand eventually converges to zero. This signifies that all the RSUs and CP1 agree on the amount of storage allocation. Although not shown, the gap between storage allocation and demand for other CP-RSU pairs also converges to zero. Fig. 2(a) and Fig. 2(b) we conclude that the proposed algorithm elicits the hidden utility and cost function of both CPs and RSUs and obtains the optimal storage allocation which attains MSW. This verifies that the proposed iterative double auction-based mechanism is incentive compatible and efficient as in Section V-C.



(a) Effect of tolerance value on the SW (b) Effect of tolerance value on the number of iteration

Fig. 3: Effect of tolerance value

Effect of the tolerance value on the social welfare: Figs. 3(a) and 3(b) show the variation of social welfare and the number of iterations with the change in the tolerance value ϵ, respectively, for different values of step size δ. Specifically, we varied the tolerance value between 0.1-0.7 and considered three different step sizes, viz., $\delta = 0.04, 0.08,$ and 0.12. We observed that, for given step size, when the tolerance value increases, the SW value at convergence deviates form the optimal SW value. This is due to the fact that, with the increase in the tolerance value, the acceptable error between the optimal SW and SW value at convergence increases. Further, with the increase in step size, we observed that the deviation of the SW value at convergence with respect to the optimal SW value increases. This is because the increase in step size allows the

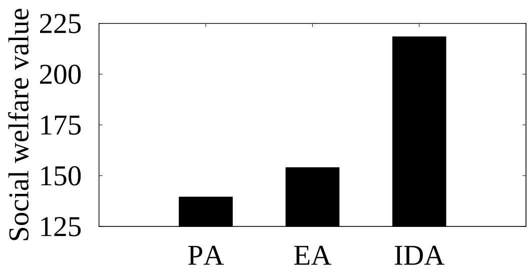
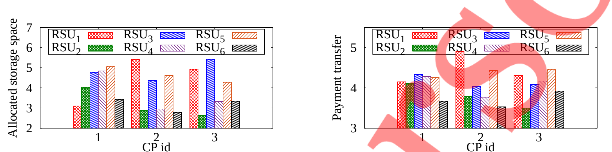


Fig. 4: Comparison of social welfare

algorithm to converge, faster which is further demonstrated in Fig. 3(b).

Comparison of social welfare: To show the effectiveness of the proposed IDA mechanism, we consider two different schemes as benchmarks. 1) *Proportional allocation* (PA): In the PA scheme, the preference of the CPs are known to the MM, and hence, the storage space of RSUs is allocated in proportion to the CPs' preference, i.e., $x_{cr} = C_r * (\frac{\theta_{cr}}{\sum_{c \in \mathcal{C}} \theta_{cr}}), \quad \forall c \in \mathcal{C}, r \in \mathcal{R}$. 2) *Equal allocation* (EA): In the EA scheme, the MM is unaware of the preference and the serving cost parameters of the CPs and RSUs, respectively. Thus, the MM allocates the storage space of RSUs equally among all the requesting CPs i.e. $x_{cr} = \frac{C_r}{|\mathcal{C}|}, \quad \forall c \in \mathcal{C}, r \in \mathcal{R}$. For performance comparison, the default values of the system parameters were considered to be the same for both the benchmark schemes and the proposed IDA mechanism.

Fig. 4 depicts the social welfare value attained by various schemes. We observe that the IDA mechanism improves the social welfare by 29.3% and 36.13% compare to PA and EA scheme, respectively. This is because unlike other schemes, IDA mechanism consider both the preference of CP and serving cost of RSU for computing the final allocation.

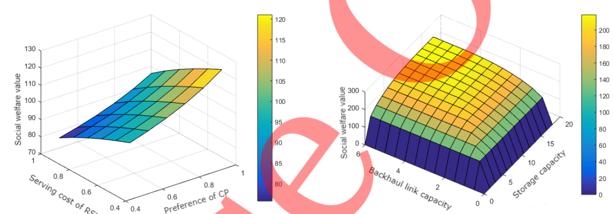


(a) Allocated storage space (b) Payment transfer

Fig. 5: Storage allocation and payment transfer at convergence

Storage allocation and payment transfer at convergence: Fig. 5(a) depicts the allocation of RSUs' storage space for different CPs. We observe that CP1 receives the maximum amount of storage space from RSU5 and obtains minimum storage space from RSU1. This is because the cost of serving CP1 is least for RSU5 and highest for RSU1 as shown in Table II. In fact, the allocation of storage space to a CP at convergence is inversely proportional to the serving cost of RSU. Further, we observe in Fig. 5(a) that RSU1 allocates maximum storage space to CP2 than other CPs. This is because the serving cost of RSU1 for CP2 is lower than other CPs in Table II. The same result can be verified from the allocation rule given in Eqn. $\boldsymbol{E_{22}}$.

Fig. 5(b) depicts the payment transfer done by the CPs for storage allocation. We observe that the payment of CP depends on both the amount of storage allocation and the serving cost of RSU. For example, CP3 pays more to RSU5 compared to RSU2. This is quite straightforward because CP3 receives larger amount of storage space from RSU5 than RSU2 (as shown in Fig. 5(a)). Interestingly, CP3 pays less to RSU1 compared to RSU5 although CP3 receives more storage space from RSU1 than RSU5. This is because of the serving cost of RSU1 is more compared to RSU5 shown in Table II.



(a) Effect of RSU serving cost and CP preference parameters (b) Effect of backhaul link capacity and RSU storage capacity

Fig. 6: Effect of system parameters on social welfare

Effect of system parameters on social welfare: In Fig. 6(a), we demonstrate the effect of CP preference and the RSU serving cost parameter on the social welfare of the system. In this case, we consider homogeneous preference and serving cost parameter, i.e. every CP has an equal preference towards each RSU and vice versa. We observe that social welfare decreases with increasing serving cost while keeping preference of CP constant. Further, with an increase in the preference of CP, social welfare value increases when the serving cost parameter is kept constant. We observe that the value of social welfare attains maximum value for higher value of CPs' preference and lower value of RSUs' serving cost parameter.

In Fig. 6(b), we demonstrate the effect of backhaul link capacity (b_{cr}) and RSU's storage capacity (S_r) on the social welfare of the system. We observe that for a fixed value of b_{cr}, the social welfare value initially increases with an increase in the RSU's storage capacity and converges eventually. This is because of the fact that the utility of CP which depends on RSU's allocated storage space, is upper bounded by the backhaul link capacity between the RSU and CP as shown in Eqn. (2). Similarly, for a given value of S_r, the backhaul capacity has a similar effect on the social welfare due to the constrained mentioned in (4). Hence, in order to improve the social welfare, we need to increase the backhaul capacity and the storage capacity simultaneously. Further, we observe that for a given backhaul capacity there exists an optimal RSU storage capacity which maximizes the social welfare of the system.

VIII. CONCLUSION

In this paper, we studied the storage allocation problem among multiple CPs in RSU-based caching networks while considering their backhaul link capacity. The problem is analyzed from the social welfare perspective and modeled as a market with multiple buyers and sellers. The utility and the cost functions of CPs and RSUs are considered as their private information. Hence, to compute the optimal storage allocation for the CP and corresponding reimbursement for the RSU an alternative optimization is formulated. Further, the iteration-based double-sided auction is employed which iteratively computes the optimal allocation and reimbursement. Furthermore, the convergence of the iterative mechanism along with its vital economical properties – incentive compatibility, individual rationality, efficiency, and budget balancing are analyzed both analytically and numerically.

In the future, we want to extend this work for the scenario where vehicular users are characterized heterogeneous content access probability. Also, we plan to implement the proposed mechanism in real testbed.

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