Assignment No.1

7th September 2020

Solutions:

1. Prove the following, giving justification for each step :

$$\neg(\exists x P(x) \wedge \forall y \neg(M(x, y))) \wedge P(a) \rightarrow \exists z M(a, z)$$

Sol.

$\rightarrow \neg(\exists x P(x) \wedge \forall y \neg(M(x, y))) \wedge P(a)$

$\rightarrow (\forall x P(x) \vee \exists y M(x, y)) \wedge P(a)$ (taking the $\neg$ sign inside)

$\rightarrow (\forall x P(x) \wedge P(a)) \vee (\exists y M(x, y) \wedge P(a))$ (distribute over $\vee$)

$\rightarrow \exists y M(a, y) \wedge P(a)$ ($(\forall x P(x) \wedge P(a))$ is false for x=a)

$\rightarrow \exists y M(a, y)$ (since both the clauses are true)

$\rightarrow \exists y M(a, z)$ (since y is a bound variable and hence dummy)

2. Use Peano's Axioms to prove the following, giving justification for each step :

$$(2 + 2) = S(S(S(S(0))))$$

Sol.

$\rightarrow (2 + 2)$

$\rightarrow S(S(0)) + S(S(0))$ (By definition $2 \equiv S(S(0))$)

$\rightarrow S(S(S(0)) + S(0))$ (since $a + S(b) = S(a + b)$)

$\rightarrow S(S(S(S(0)) + 0))$ (since $a + S(b) = S(a + b)$)

$\rightarrow S(S(S(S(S(0)))))$ (since $a + 0 = a$)