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   - Maximum Sum Subarray by brute force
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Divide and Conquer Strategy

1. Given a problem, identify a small number of smaller subproblems of the same type and of similarly sizes
2. Solve each subproblem recursively (the smallest possible size of a subproblem is a base-case)
3. Combine these solutions into a solution for the main problem

$$T(n) = \begin{cases} T_{\text{Divide}} + \sum_{P_i} T(|P_i|) + T_{\text{Combine}} \\ T_{\text{Base}} \end{cases}$$

Polynomial multiplication

- $C(x) = A(x)B(x) = \sum_{i=0}^{2n-2} c_i x_i,\ c_i = \sum_{0 \le j, i-j \le n-1} a_j b_{i-j}$
  [Note the convolution of the coefficients]
- Time complexity of this scheme is:

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- Time complexity of this scheme is: $O(n^2)$, actually $\Theta(n^2)$

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float polyMul(float *A, *B, *C, int deg) {
  int j, k;
  for (j=0; j<=2*deg-2; j++) C[j] = 0;
  for (j=0; j<=deg-1; j++)
    for (k=0; k<=deg-1; k++)
      C[j+k] += A[j] * B[k];
}
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- Can we do better than $\Theta(n^2)$, how could we do that?
- Since the time grows with the degree, can we compute the product multiplying smaller polynomials?

Polynomial multiplication by DC

- $A(x) = A_L(x) + x^t A_H(x)$
  $A_L(x) = a_0 + a_1 x + \ldots + a_{t-1} x^{t-1}$,
  $A_H(x) = a_t + a_{t+1} x + \ldots + a_{n-1} x^{n-1-t}$
- $t = \left\lfloor \frac{n}{2} \right\rfloor$, so that both $A_L(x)$ and $A_H(x)$ are nearly equal
- Ideally, n is a power of 2, $n = 2^d, d \geq 0$
- Similarly, $B(x) = B_L + x^t B_H$, where $B_L \equiv B_L(x)$ and $B_H \equiv B_H(x)$
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- Smaller polynomials recursively multiplied until the degree reduces to 0, when the coefficients are directly multiplied
- $T_{\text{mulHL}}(n) = \begin{cases} a & [n = 1] \\ 4T_{\text{mulHL}}\left(\frac{n}{2}\right) + bn + c & [n > 1, n = 2^d, d \geq 0] \end{cases}$

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- No improvement, but why? There are too many sub-problems

Polynomial multiplication by Karatsuba DC

- $C(x) = A(x)B(x) = x^{2t}A_HB_H + x^t(A_HB_L + A_LB_H) + A_LB_L$

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- Store and reuse A_HB_H and A_LB_L computed earlier
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- Solution (by standard methods): $T_{\text{mulHL}}(n) \in \Theta\left(n^{\log_2 3}\right)$, $\log_2 3 = 1.58496\ldots$

Maximum Sum Subarray by brute force

- You are given a one dimensional array that may contain both positive and negative integers
- Find the sum of contiguous subarray of numbers which has the largest sum, e.g.: {-3, -6, **7, -1, -3, 2, 5**, -7}
- Time taken is for finding sum of intervals and computing their max
- $T(n) = \sum_{j=n}^{j=1} C(n-j+1, n) + \beta n$, where $C(i, n)$ is the cost of summing i intervals each having $n-i+1$ elements
- $C(i, n) = \alpha i(n-i+1)$ (naive)
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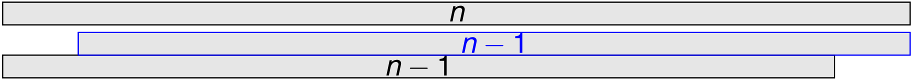

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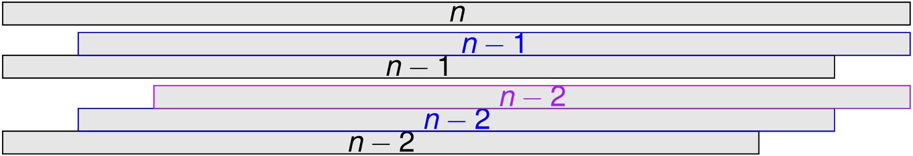

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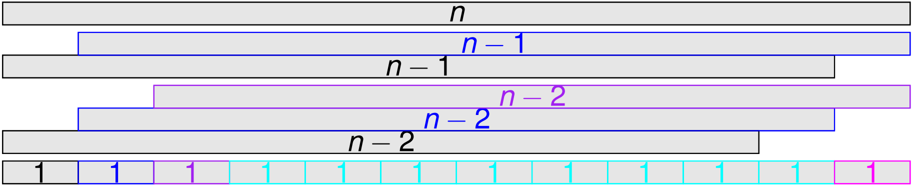

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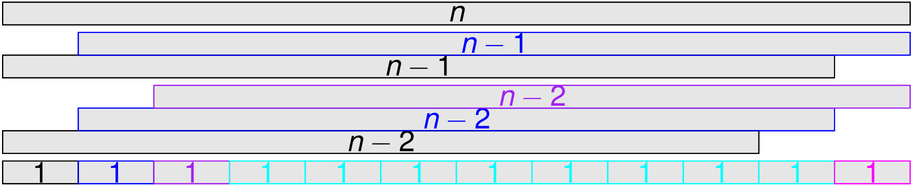

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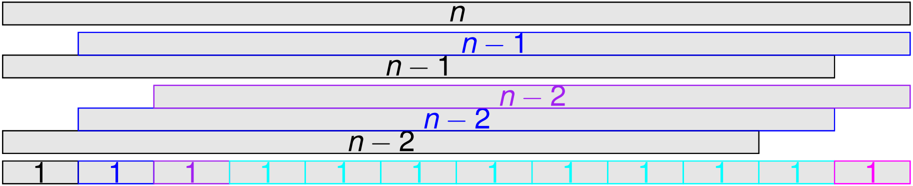

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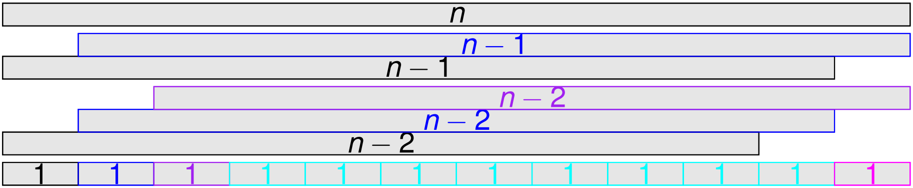

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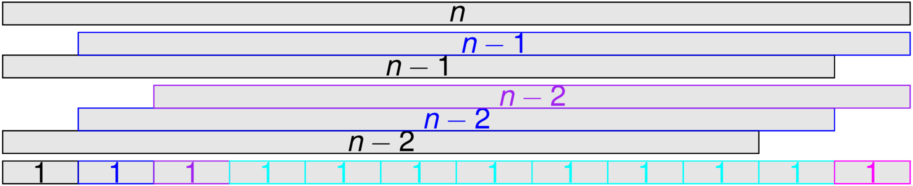

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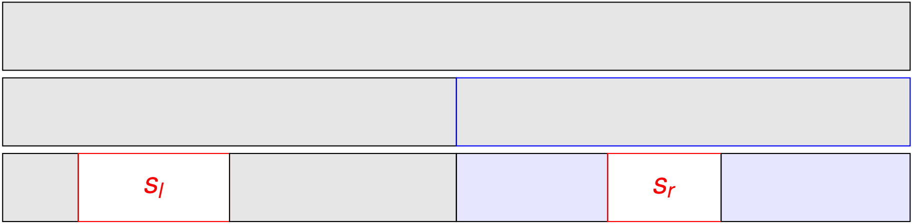

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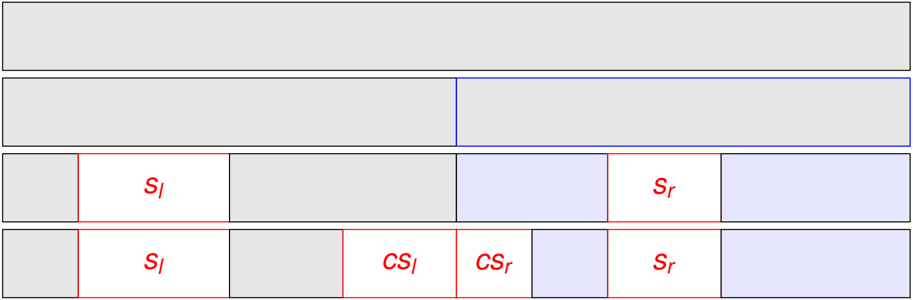

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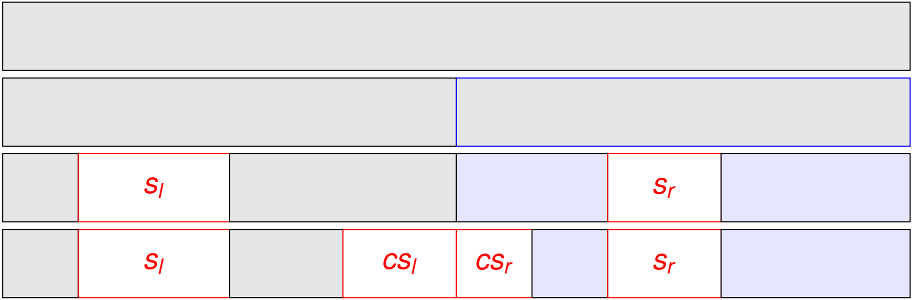

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Maximum Sum Subarray by DC (contd.)

```
int maxSumSubarray(int A[], int l, int h) {
  // base case when A has only one element
  if (l == h) return A[l];
  // now the recursive steps
  int m = (l + h)/2; // divide A at the middle
  // need to compute max of
  // the maxSumSubarray of each part and
  // the max sum in the interval containing A[m]



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  return max(
    maxSumSubarray(A, l, m),
    maxSumSubarray(A, m+1, h),
    maxCrossSum(A, l, m, h));
}
```

Computing maxCrossSum

```
int maxCrossSum(int A[], int l, int m, int h) {
  // at least A[m] and A[m+1] are present, h>l
  int sum = A[m]; int lSum = A[m];
  for (int i = m-1; i >= l; i--) {
    sum = sum + A[i];
    if (sum > lSum) lSum = sum;
  }
  sum = A[m+1]; int rSum = A[m+1];
  for (int i = m+2; i <= h; i++) {
    sum = sum + A[i];
    if (sum > rSum) rSum = sum;
  }
  return lSum + rSum;
}
```

Matrix multiplication by DC

- Given, $n \times n$ matrices $A = (a_{ij})$, $B = (b_{ij})$, product $C = AB$ is defined as $c_{ij} = \sum_{k=0}^{n} a_{ik} b_{kj}$
- This way, computation of each c_{ij} takes $\Theta(n)$ time
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- Application of DC can be considered
- $A = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix}$ $B = \begin{pmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{pmatrix}$
  $C = \begin{pmatrix} A_{11}B_{11} + A_{12}B_{21} & A_{11}B_{12} + A_{12}B_{22} \\ A_{21}B_{11} + A_{22}B_{21} & A_{21}B_{12} + A_{22}B_{22} \end{pmatrix}$
- Time complexity: $T(n) =$

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  $C = \begin{pmatrix} A_{11}B_{11} + A_{12}B_{21} & A_{11}B_{12} + A_{12}B_{22} \\ A_{21}B_{11} + A_{22}B_{21} & A_{21}B_{12} + A_{22}B_{22} \end{pmatrix}$
- Time complexity: $T(n) = 8T\left(\frac{n}{2}\right) + \Theta(n^2)$
- Solution (by standard methods): $\Theta(n^3)$, so no benefit

Matrix multiplication by DC (contd.)

- Strassen's matrix multiplication
  algorithm works by first defining 7 intermediate matrices as follows:

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- Solution (by standard methods): $\Theta(n^{2.80735\cdots})$
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Practice problems

- Given a sorted array in which all elements appear twice (together) and one element appears only once, locate that element
- An array of n points in the plane is given; find out the closest pair of points in the array
- Find the largest rectangular area possible in a given histogram where the largest rectangle can be made of a number of contiguous bars. For simplicity, assume that all bars have same width of 1 unit
- Given a $n \times n$ board where n is a power of 2 with minimum value as 2, with one missing cell (of size 1×1) at a known location, fill the board using L shaped tiles An L shaped tile is a 2×2 square with one cell (of size 1×1) missing
- There are two sorted arrays A and B of size n each, find the median of the array obtained after merging the above 2 arrays
- Check if a given integer p appears more than $\frac{n}{2}$ times in a sorted array of n integers