1 Greedy Algorithms

1.1 Non-cut edge

Given an connected, undirected graph $G = (E, V)$, find whether it has a non-cut edge. A *cut edge* is an edge whose removal from G makes it disconnected. Any other edge is a *non-cut edge*.

1.2 Minimum edges

Show that if an undirected graph $G = (V, E)$ with n vertices has k components, then it has at least $n - k$ edges. Recall that $S \subseteq V$ is a *component* in G if (i) there is a path in G between every two vertices of S and (ii) there is no path in G between any vertex of S and any vertex of $V \setminus S$.

1.3 Change in MST and shortest paths

Consider an undirected graph $G = (V, E)$ with edge weights $w_e \geqslant 0$. Suppose that you have computed a minimum spanning tree of G, and that you have also computed shortest paths to all nodes from a particular node $s \in V$. Now suppose each edge weight is increased by 1: the new weights are $w'_e = w_e + 1$.

(a) Does the minimum spanning tree change? Give an example where it changes or prove it cannot change.

(b) Do the shortest paths change? Give an example where they change or prove they cannot change.

1.4 Interval-graph coloring

Suppose that we have n activities to schedule among a large number of lecture halls, where any activity can take place in any lecture hall. We wish to schedule all the activities using as few lecture halls as possible. Give an efficient greedy algorithm to determine which activity should use which lecture hall. An activity i is specified by its start and end times, a_i and b_i.

This problem is also known as the *interval-graph coloring* problem. We can create an *interval graph* whose vertices are the given activities and whose edges connect incompatible activities. The smallest number of colors required to color every vertex so that no two adjacent vertices have the same color corresponds to finding the fewest lecture halls needed to schedule all of the given activities.

1.5 Minimize gas fill

Prof Midas drives an automobile from Newark to Reno along Interstate 80. His car's gas tank, when full, hold enough gas to travel n miles, and his map gives the distances between gas stations on his route. The professor wishes to make as few gas stops as possible along the way. Give an efficient algorithm by which he can determine at which gas stations he should stop, and prove that your strategy yields an optimum solution.

1.6 Minimize unit intervals

Describe an efficient algorithm that, given a set $\{x_1, x_2, \ldots, x_n\}$ of points on the real line, determines the smallest set of unit-length closed intervals that contains all of the given points. Argue that your algorithm is correct.