Construction of 3D Orthogonal Cover of a Digital Object

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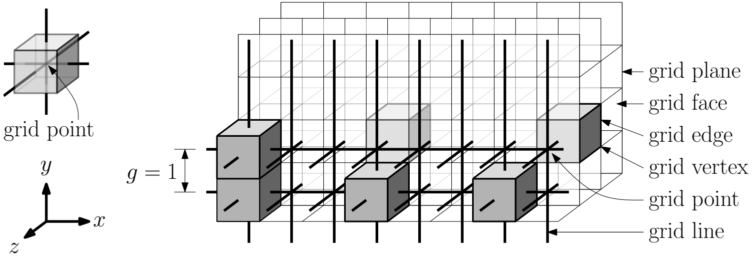
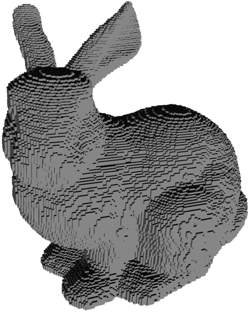
Theoretical Basis

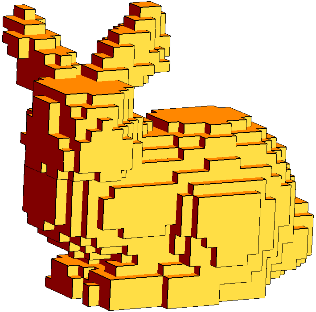


3D digital space and 26N [14].

Input & Output



$n = 58605$

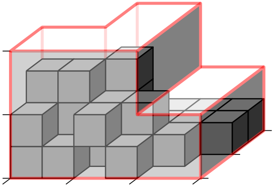


$g = 7 : n_v = 1031, n_f = 588$

Objective

Given the **3D object** A imposed on the **grid** $\mathbb{G}$, the problem is to construct its **orthogonal polytope** $P(A, \mathbb{G})$, such that:

- each point $p \in A$ lies inside $P(A, \mathbb{G})$;
- each vertex of $P(A, \mathbb{G})$ is a grid vertex;
- each edge of $P(A, \mathbb{G})$ is parallel to one of the coordinate axes;
- each face of $P(A, \mathbb{G})$ lies on some grid plane;
- volume of $P(A, \mathbb{G})$ is minimized.



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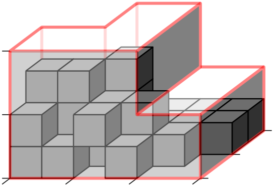

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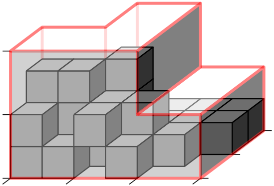

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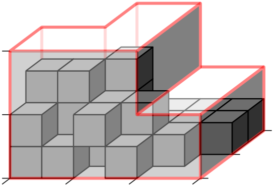

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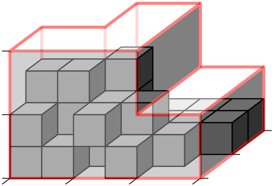

Data structures

- **Grid** $\mathbb{G}$: 3D array of UGCs (8 vertices, 12 edges, 6 faces)
- **Doubly connected edge list (DCEL)** [1, 18]:
  - **vertex list** $V = |vid|(x, y, z)|$
  - **edge list** $E = |eid|fid|start\ vid|next|prev|pair|$
  - **face list** $F = |fid|eid|plane|color|$

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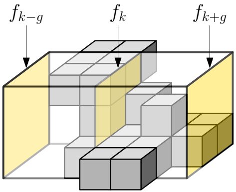
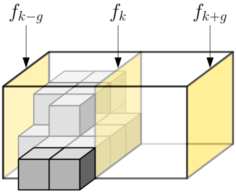
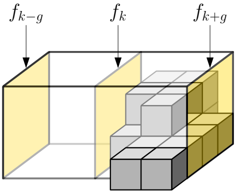
Eligibility of a face ($g = 3$)



$\text{ELIGIBLEFACE}(f_k) = \text{FALSE}$

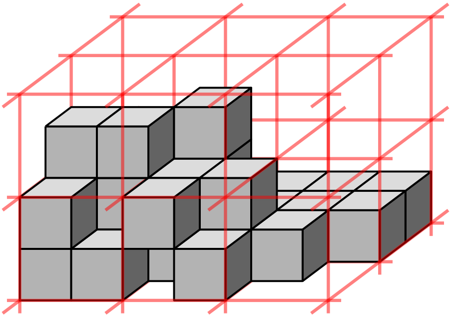


$\text{ELIGIBLEFACE}(f_k) = \text{TRUE}$

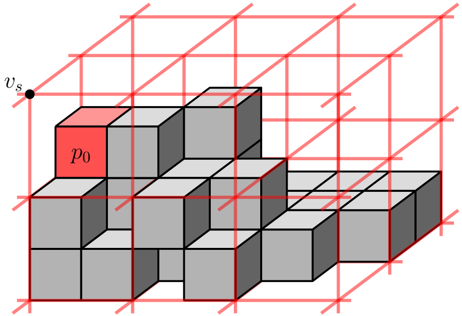


$\text{ELIGIBLEFACE}(f_k) = \text{TRUE}$

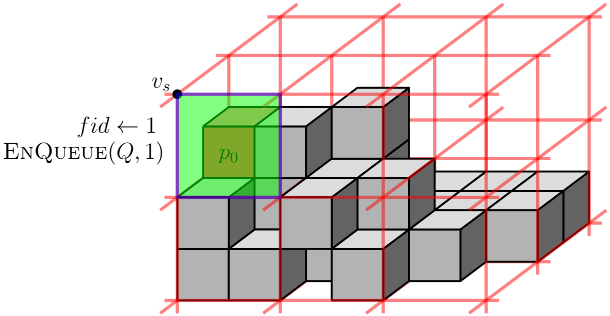
DCEL Construction



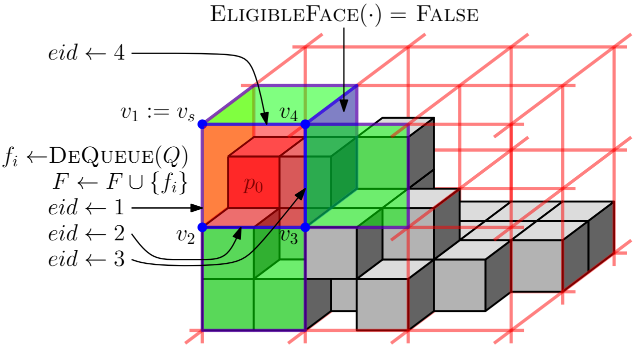
DCEL Construction



DCEL Construction



DCEL Construction



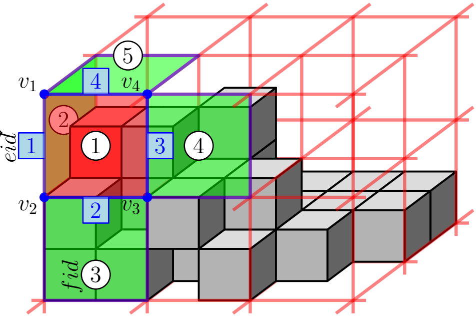
DCEL Construction

$$F = \{\ f_1 = \ |1|1|xy|\ \}$$

$$E = \left\{ \begin{array}{l} e_1 = \ |1|1|1|2|4|1|, \\ e_2 = \ |2|1|2|3|1|2|, \\ e_3 = \ |3|1|3|4|2|3|, \\ e_4 = \ |4|1|4|1|3|4| \end{array} \right\}$$

$$V = \{1, 2, 3, 4\}$$

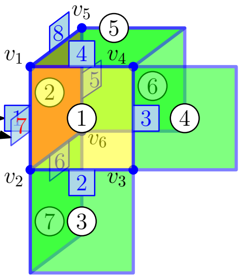
$$Q = \langle 2, 3, 4, 5 \rangle$$



f_1 dequeued

DCEL Construction

pairing of half-edges
$e_1(v_1, v_2) \in f_1$
and
$e_7(v_2, v_1) \in f_2$
not done for
non-coplanarity



f_2 dequeued

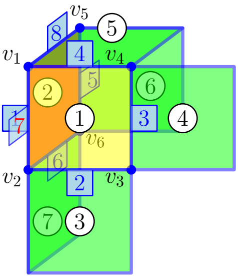
DCEL Construction

$$F = \left\{ \begin{array}{ll} f_1 = & |1|1|xy|, \\ \color{red}{f_2 =} & |2|5|yz| \end{array} \right\}$$

$$E = \left\{ \begin{array}{ll} e_1 = & |1|1|1|2|4|\color{red}{1}|, \\ e_2 = & |2|1|2|3|1|2|, \\ e_3 = & |3|1|3|4|2|3|, \\ e_4 = & |4|1|4|1|3|4|, \\ \color{red}{e_5 =} & |5|2|5|6|8|5|, \\ \color{red}{e_6 =} & |6|2|6|7|5|6|, \\ \color{red}{e_7 =} & |7|2|2|8|6|\color{red}{7}|, \\ \color{red}{e_8 =} & |8|2|1|5|7|8| \end{array} \right\}$$

$$V = \{1, 2, 3, 4, \color{red}{5, 6}\}$$

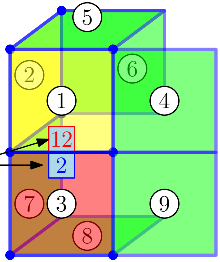
$$Q = \langle 3, 4, 5, \color{red}{6, 7} \rangle$$



f_2 dequeued

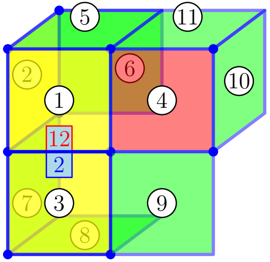
DCEL Construction

pairing of half-edges $e_2 \in f_1$ and $e_{12} \in f_3$, since f_1 and f_3 are coplanar



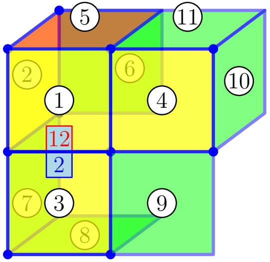
$Q = \langle 4, 5, 6, 7, 8, 9 \rangle$

DCEL Construction



$Q = \langle 5, 6, 7, 8, 9, 10, 11 \rangle$

DCEL Construction



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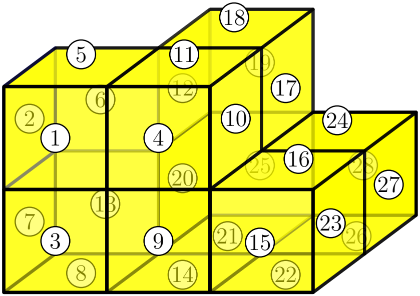
DCEL Construction

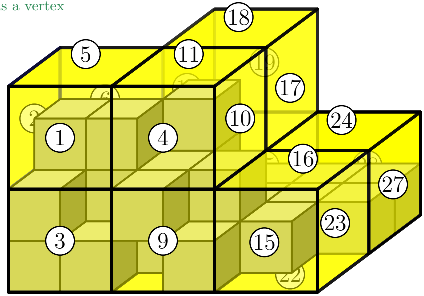


$Q = \langle \ \rangle$

DCEL Construction

Algorithm ORTHOCOVER3D$(A, \mathbb{G})$

$v_s \leftarrow (i_s = \lfloor i_0/g \rfloor \times g,\ j_s = \lceil j_0/g \rceil \times g,\ k_s = \lceil k_0/g \rceil \times g)$
pick the front face f_s of the UGC having v_s as a vertex
$n_f \leftarrow\ n_e \leftarrow n_v \leftarrow 0$
$fid[f_s] \leftarrow n_f \leftarrow n_f + 1$
$color[f_s] \leftarrow$ GRAY
ENQUEUE(Q, f_s)
while Q is not empty
  $f_i \leftarrow$ DEQUEUE(Q)
  for each edge $e_{ij} \in f_i$
    $eid[e_{ij}] \leftarrow n_e \leftarrow n_e + 1$
    $face[e_{ij}] \leftarrow fid[f_i]$
    if $start[e_{ij}] \notin V$
      $vid[start[e_{ij}]] \leftarrow n_v \leftarrow n_v + 1$
      $V \leftarrow V \cup \{start[e_{ij}]\}$
    assign $start[e_{ij}],\ next[e_{ij}],\ prev[e_{ij}]$
    ▷ from array $\mathbb{G}$
    $pair[e_{ij}] \leftarrow eid[e_{ij}]$
    $E \leftarrow E \cup \{e_{ij}\}$
    for each face f_k incident on
    $(start[e_{ij}], start[next[e_{ij}]])$
      if ELIGIBLEFACE(f_k) = TRUE
        if $color[f_k]$ = WHITE
          $fid[f_k] \leftarrow n_f \leftarrow n_f + 1$
          $color[f_k] \leftarrow$ GRAY
          EnQueue(Q, f_k)
        else if $color[f_k]$ = BLACK **and** f_i and f_k are coplanar
          $pair[e_{ij}] \leftarrow eid[\bar{e}_{ij}], pair[\bar{e}_{ij}] \leftarrow eid[e_{ij}]$ ▷ $\bar{e}_{ij} \in f_k$
  $edge[f_i] \leftarrow eid[e_{i1}]$ ▷ e_{i1} is the 1st edge of f_i
  $color[f_i] \leftarrow$ BLACK
  $F \leftarrow F \cup \{f_i\}$
MERGEFACE(F, E)



Face Merging

starting with f_1:

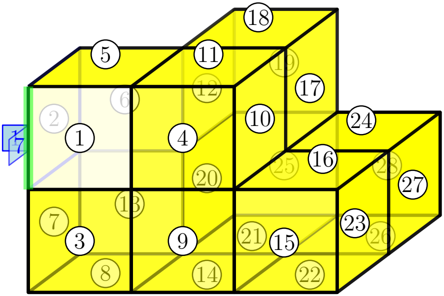
init: $count \leftarrow 4$;

f_1 not merged with f_2,

as $pair[e_1] = e_1$;

so $e_2 = next[e_1]$ and

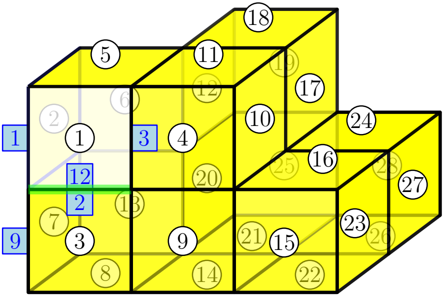
$count \leftarrow count - 1 = 3$.



Face Merging

f_3 merged with f_1,
as $pair[e_2] = e_{12} \neq e_2$;
$next[prev[e_2]] =\leftarrow next[e_{12}]$,
or, $next[e_1] \leftarrow e_9$;
$next[prev[e_{12}]] \leftarrow next[e_2]$,
or, $next[e_{11}] \leftarrow e_3$;
$face[e_{12}] \leftarrow f_1$;
f_3 deleted from F;
e_2, e_{12} deleted from E;
$count \leftarrow count + 2 = 5$;

repeat with next edge e_9
until $count = 0$.



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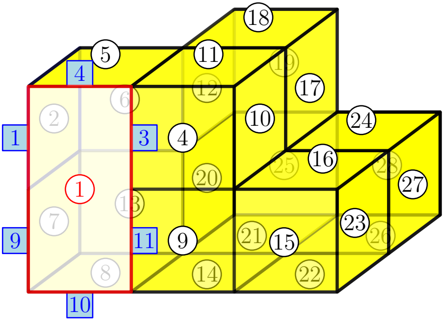
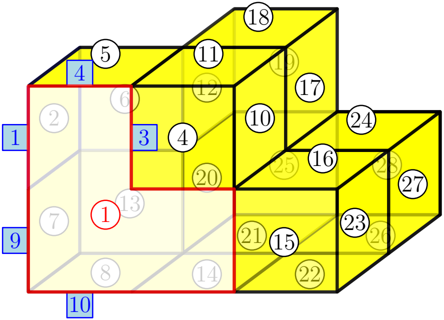


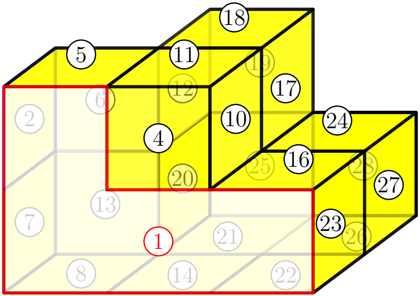
Face Merging

f_7 not merged with f_1,
$count \leftarrow count - 1 = 4$;
f_8 not merged with f_1,
$count \leftarrow count - 1 = 3$;
f_9 merged with f_1,
$count \leftarrow count + 2 = 5$;



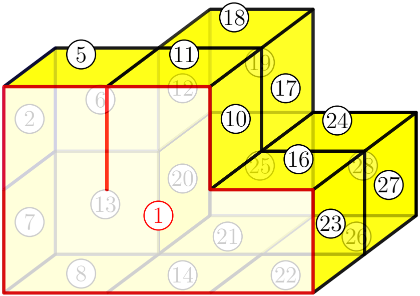
Face Merging

f_{14} not merged with f_1,
$count \leftarrow count - 1 = 4$;
f_{15} merged with f_1,
$count \leftarrow count + 2 = 6$;



Face Merging

f_{22} not merged with f_1,
$count \leftarrow count - 1 = 5$;
f_{23} not merged with f_1,
$count \leftarrow count - 1 = 4$;
f_{16} not merged with f_1,
$count \leftarrow count - 1 = 3$;
f_4 merged with f_1,
$count \leftarrow count + 2 = 5$;



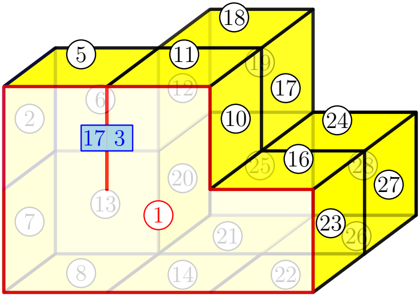
Face Merging

f_{10} not merged with f_1,
$count \leftarrow count - 1 = 4$;
f_{11} not merged with f_1,
$count \leftarrow count - 1 = 3$;

$face[e_{17}] = face[e_3]$,
$count \leftarrow count - 1 = 2$;

$face[e_3] = face[e_{17}]$,
$count \leftarrow count - 1 = 1$;

f_5 not merged with f_1,
$count \leftarrow count - 1 = 0$;



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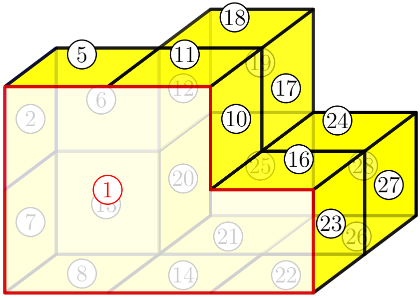
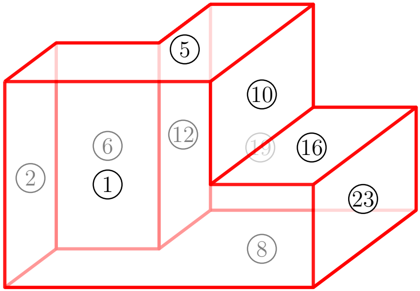


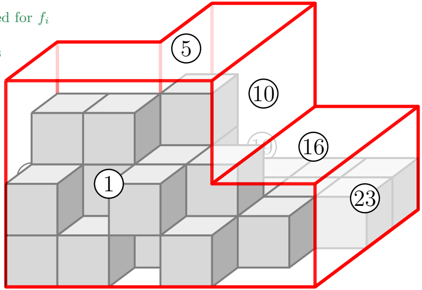
Face Merging



Face Merging

Procedure MergeFace(F, E)

01. **for** each face $f_i \in F$
02. $e_{ij} \leftarrow edge[f_i]$ ▷ 1st edge of $f_i \in F$
03. $count \leftarrow 4$ ▷ number of edges to be visited for f_i
04. **do**
05. **if** $eid[e_{ij}] \neq pair[e_{ij}]$ ▷ $pair[e_{ij}]$ exists
06. $\bar{e}_{ij} \leftarrow pair[e_{ij}]$
07. $f_k \leftarrow face[\bar{e}_{ij}]$
08. $next[prev[e_{ij}]] \leftarrow next[\bar{e}_{ij}]$
09. $next[prev[\bar{e}_{ij}]] \leftarrow next[e_{ij}]$
10. $e_t \leftarrow next[prev[e_{ij}]]$
11. $face[\bar{e}_{ij}] \leftarrow fid[f_i]$
12. $F \leftarrow F \setminus \{f_k\}$
13. $E \leftarrow E \setminus \{e_{ij}, \bar{e}_{ij}\}$
14. $e_{ij} \leftarrow e_t$
15. $edge[f_i] \leftarrow eid[e_{ij}]$
▷ 1st edge
of the modified face f_i
16. $count \leftarrow count + 2$
▷ 4 edges added to
& 2 edges removed from f_i
17. **else** ▷ $pair[e_{ij}]$ does not exist
18. $e_{ij} \leftarrow next[e_{ij}]$
19. $count \leftarrow count - 1$
20. **while** $count \neq 0$
21. **return** (F, E)



Time Complexity

For each UGC, at most five faces.
DCEL complexity (#faces, #edges, #vertices) $\leqslant O(n/g^3)$.

Best Case:
Eligibility of a face $= O(1)$ time.
Min #UGCs $= O(n/g^3)$.
$\Rightarrow$ DCEL time $= O(n/g^3) \times O(1) = \mathbf{O(n/g^3)}$.
Max #faces on the surface $O(n/g^3)$.
$\Rightarrow$ MergeFace time $= \mathbf{O(n/g^3)}$.

Worst Case:
Eligibility of a face $= O(g^3)$ time.
Max #UGCs $= O(n/g)$.
$\Rightarrow$ DCEL time $= O(n/g) \times O(g^3) = \mathbf{O(ng^2)}$.
Maximum number of faces $= O(n/g)$.
$\Rightarrow$ MergeFace time $= \mathbf{O(n/g)}$.

In practice: Tends towards the **best case**.

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DCEL complexity (#faces, #edges, #vertices) $\leqslant O(n/g^3)$.

Best Case:
Eligibility of a face $= O(1)$ time.
Min #UGCs $= O(n/g^3)$.
$\Rightarrow$ DCEL time $= O(n/g^3) \times O(1) = \mathbf{O(n/g^3)}$.
Max #faces on the surface $O(n/g^3)$.
$\Rightarrow$ MergeFace time $= \mathbf{O(n/g^3)}$.

Worst Case:
Eligibility of a face $= O(g^3)$ time.
Max #UGCs $= O(n/g)$.
$\Rightarrow$ DCEL time $= O(n/g) \times O(g^3) = \mathbf{O(ng^2)}$.
Maximum number of faces $= O(n/g)$.
$\Rightarrow$ MergeFace time $= \mathbf{O(n/g)}$.

In practice: Tends towards the **best case**.

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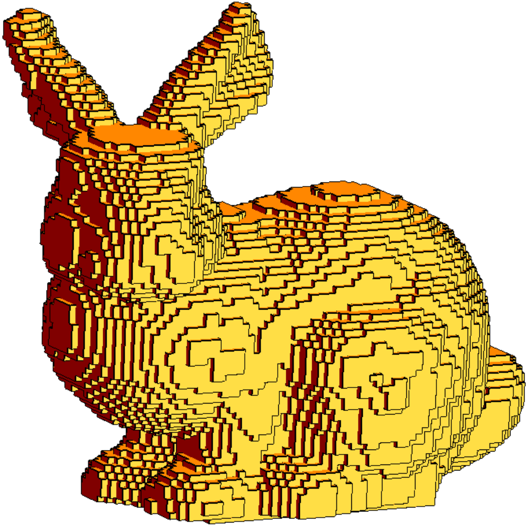
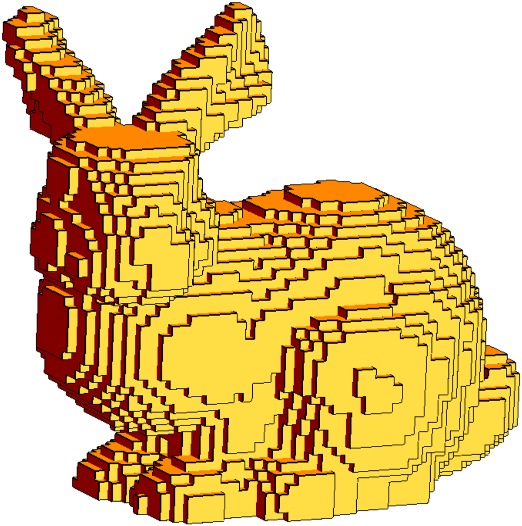
In practice: Tends towards the **best case**.

Results: Stanford bunny



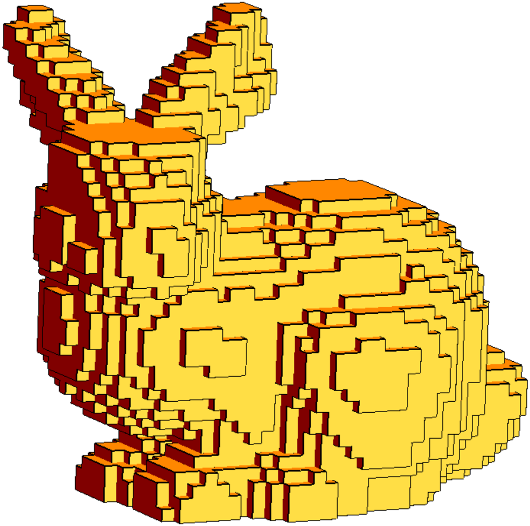
$g = 2$
$n_v = 9090$
$n_e = 43458$
$n_f = 5529$

Results: Stanford bunny



$g = 3$
$n_v = 3987$
$n_e = 19134$
$n_f = 2367$

Results: Stanford bunny



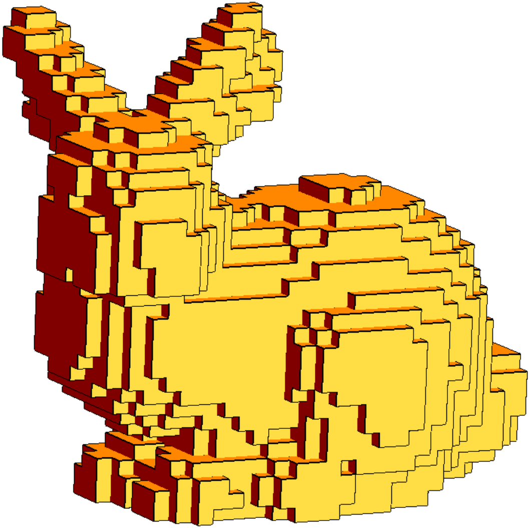
$g = 4$

$n_v = 2335$

$n_e = 11164$

$n_f = 1383$

Results: Stanford bunny



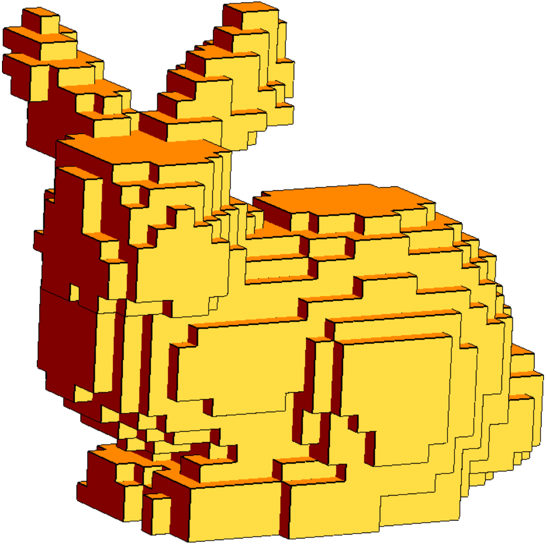
$g = 5$

$n_v = 1533$

$n_e = 7358$

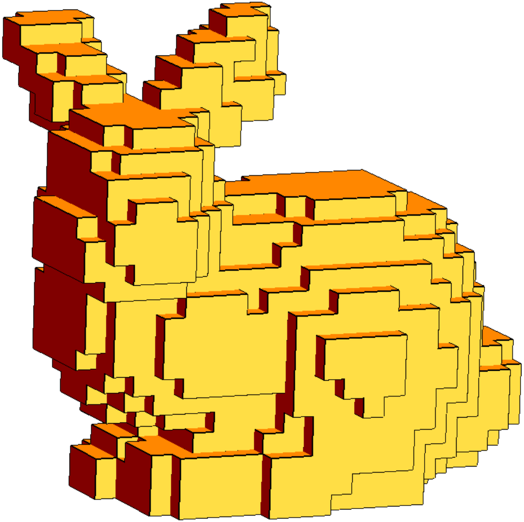
$n_f = 895$

Results: Stanford bunny



$g = 6$
$n_v = 1031$
$n_e = 5042$
$n_f = 588$

Results: Stanford bunny



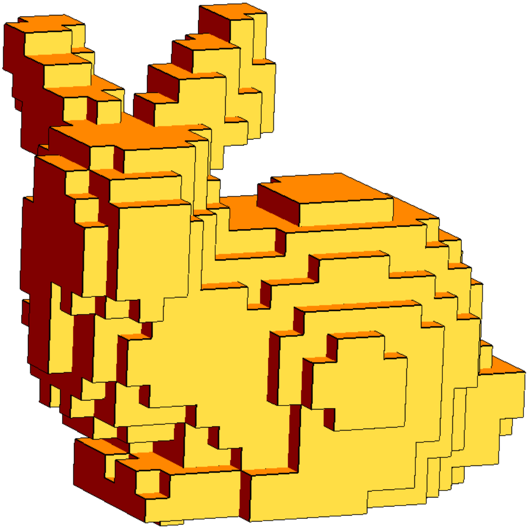
$g = 7$

$n_v = 775$

$n_e = 3894$

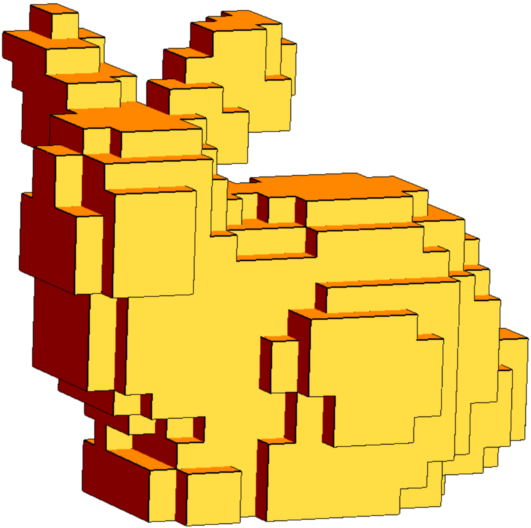
$n_f = 455$

Results: Stanford bunny



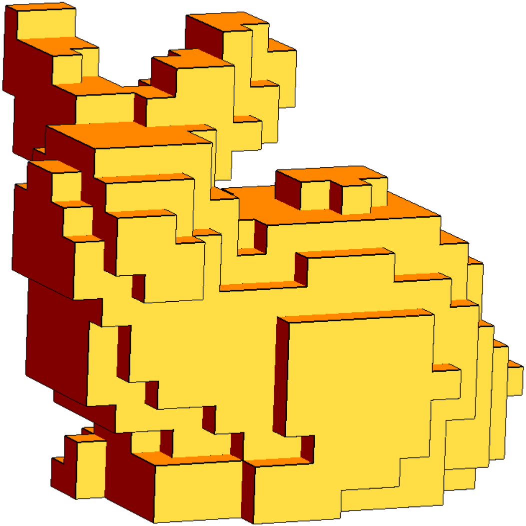
$g = 8$
$n_v = 636$
$n_e = 2972$
$n_f = 359$

Results: `Stanford bunny`



$g = 9$
$n_v = 502$
$n_e = 2440$
$n_f = 287$

Results: Stanford bunny



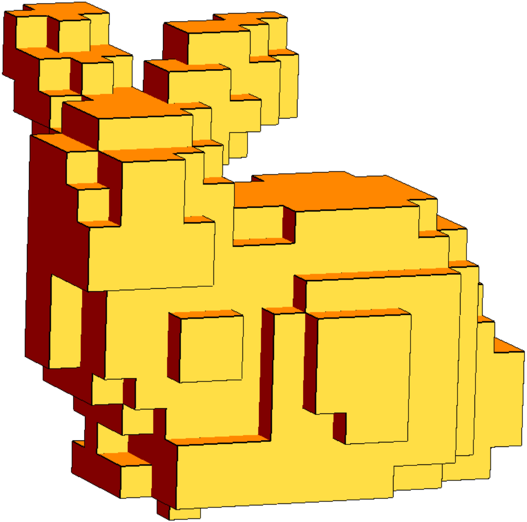
$g = 10$

$n_v = 426$

$n_e = 2016$

$n_f = 243$

Results: Stanford bunny



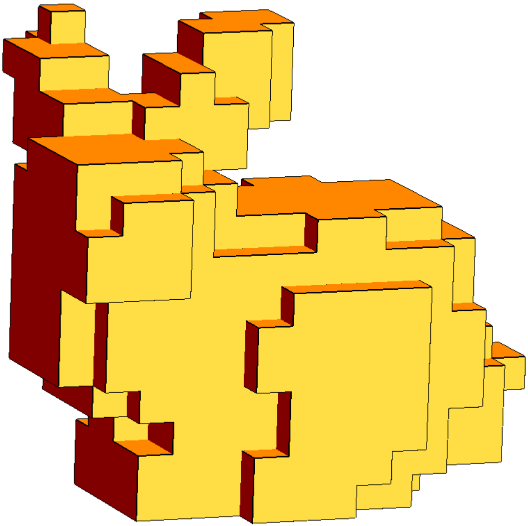
$g = 11$

$n_v = 340$

$n_e = 1696$

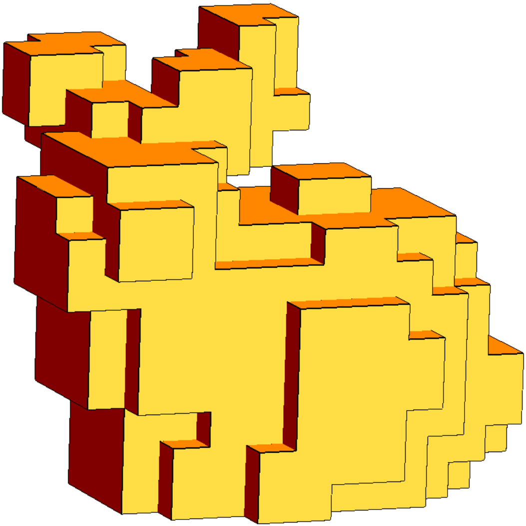
$n_f = 204$

Results: Stanford bunny



$g = 12$
$n_v = 296$
$n_e = 1398$
$n_f = 173$

Results: Stanford bunny



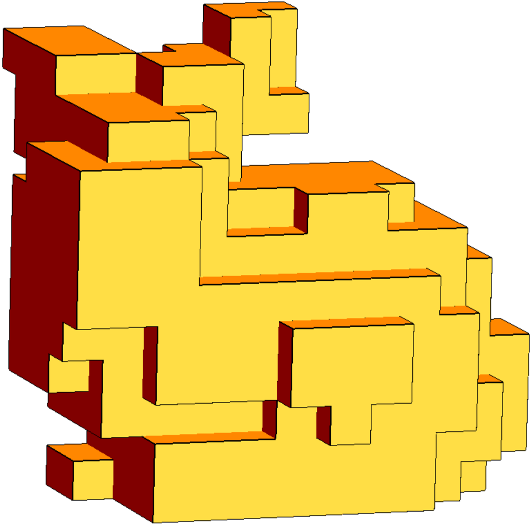
$g = 13$

$n_v = 245$

$n_e = 1246$

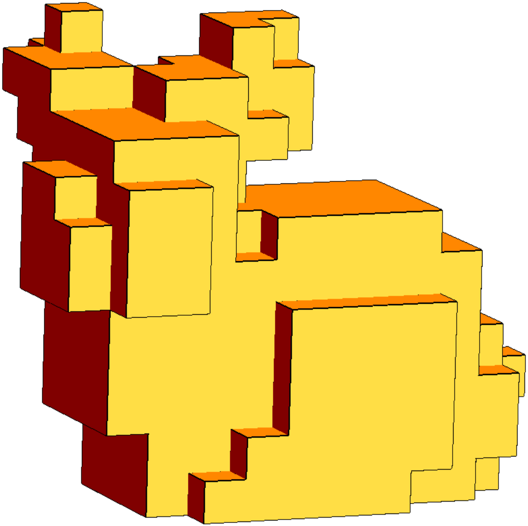
$n_f = 140$

Results: `Stanford bunny`



$g = 14$
$n_v = 210$
$n_e = 1094$
$n_f = 125$

Results: `Stanford bunny`



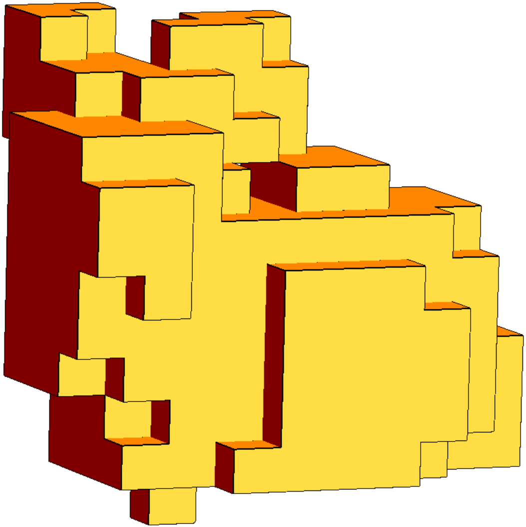
$g = 15$

$n_v = 192$

$n_e = 948$

$n_f = 118$

Results: Stanford bunny



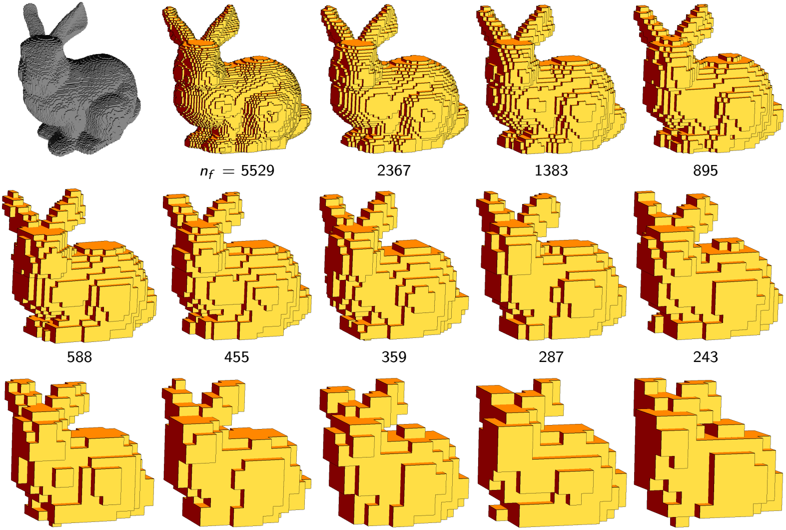
$g = 16$

$n_v = 179$

$n_e = 856$

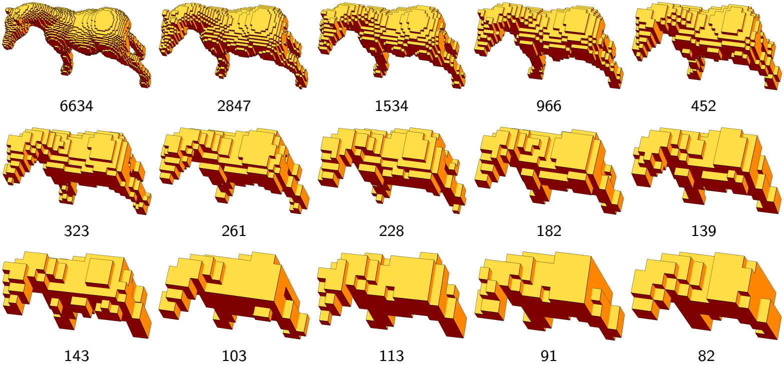
$n_f = 105$

Results: Stanford bunny

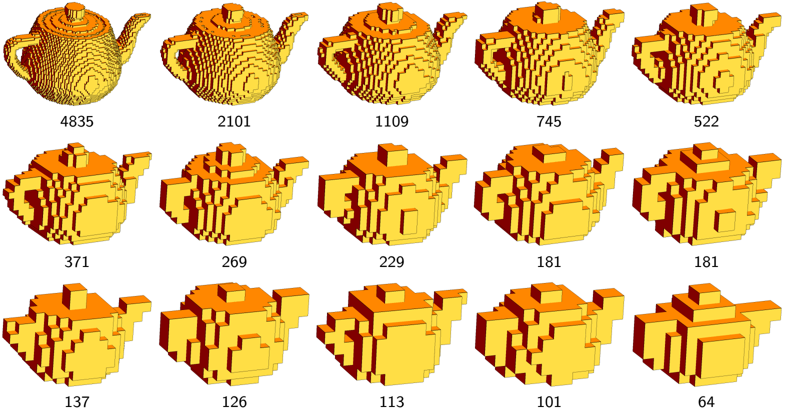


n_f = 5529, 2367, 1383, 895, 588, 455, 359, 287, 243, 204, 173, 140, 125, 105

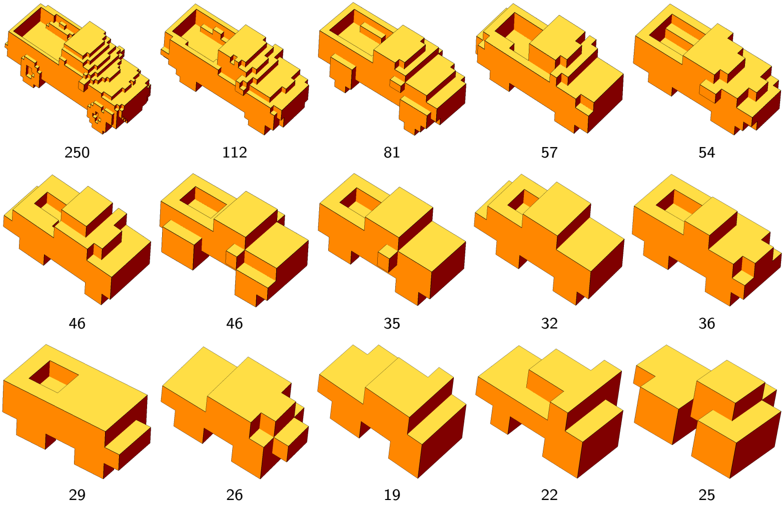
Results: horse ($g = 2, 3, \ldots, 16$)



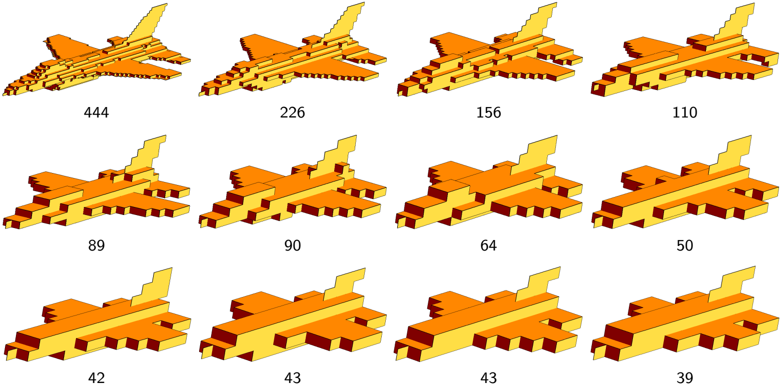
Results: teapot ($g = 2, 3, \ldots, 16$)



Results: `pickup van` ($g = 2, 3, \ldots, 16$)



Results: `airplane` $(g = 2, 3, \ldots, 13)$



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Thank You