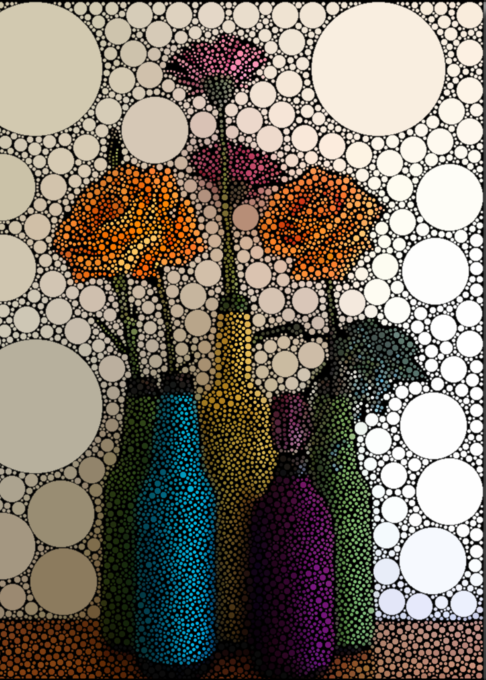


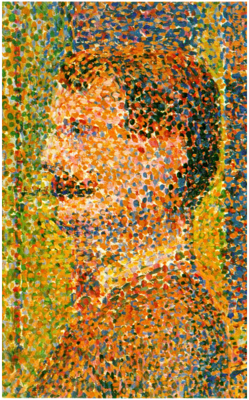
Digital Circlism

as Algorithmic Art

S. De
P. Bhowmick

Background

According to artists and critics, it's a fusion of *pop art* and *pointillism*[a]



[a] Chipp, H.B. 1996: *Theories of Modern Art*

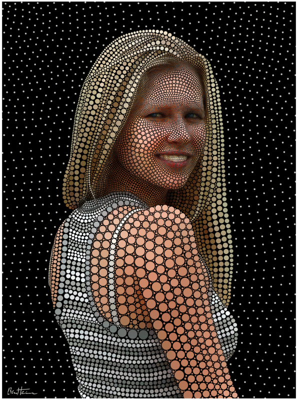
Background

First introduced by *Edward C. Stresino* in 1985[a]

[a] http://www.circlism.com

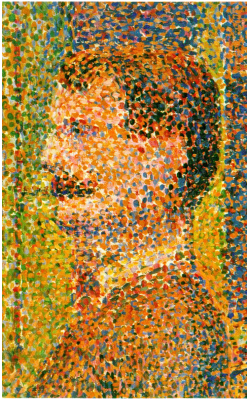
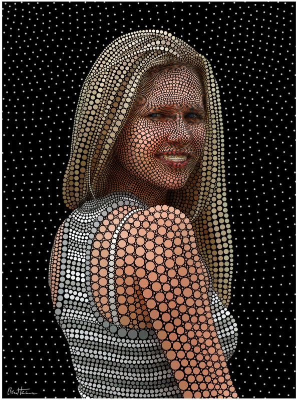
Background

Digital artworks by
Ben Heine
since 2010[a]



[a] http://benheine.deviantart.com/gallery/30782139, http://www.flickr.com/photos/benheine/sets/72157623553428960

Background

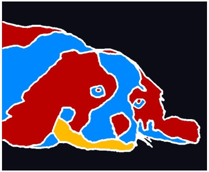
Pointillism	Pop art	Digital circlism
		
Georges Seurat (France, 1859-1891)	*Romero Britto* (Brazil, b. 1963)	*Ben Heine* (Ivory Coast, b. 1983)

Our Algorithm

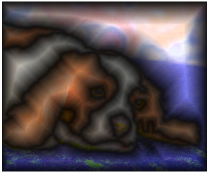
Basic Steps

input

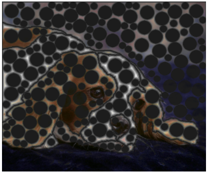
segmented



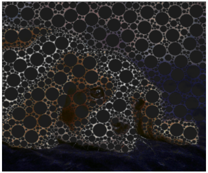
parsed



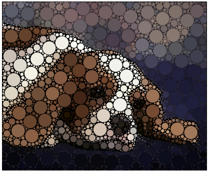
EDT



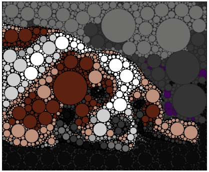
$r = 20, 10$



$r = 20, 10, 5, 2$

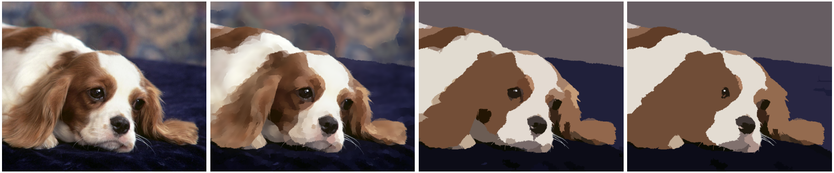


coloring



Macbeth Luv

Mean shift segmentation



Input $(h_s, h_r) = (7, 6)$ $(11, 10)$ $(14, 13)$

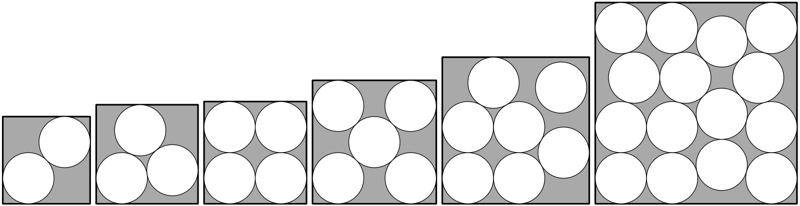
Two types of bandwidth parameters:[a,b]

- h_s in spatial domain $(d = 2)$
- h_r in CIELUV color subspace $(d = 3)$

[a] Comaniciu & Meer: Mean shift - A robust approach toward feature space analysis. IEEE PAMI (2002)

[b] Fairchild: Color Appearance Models. Addison-Wesley (1998)

Circle packing (1)



2	3	4	5	7	15
$\rho = 0.53901$	0.60965	0.78540	0.67377	0.66931	0.76206

Packing density

$\rho =$ proportion of the region covered by packing circles—forms the *maximization criterion*.

Circle packing

Solution to date: Only for packing inside geometric primitives like *squares*, *circles*, *triangles*, etc.

Difficulty: Many packing problems are *NP-hard*.[c,d]

Our work: Packing circles in *discrete space* instead of real space, where the circles are defined by a *finite and small set of radii*.

[c] Demaine *et al.*: Circle packing for origami design is hard. arxiv.org (2010)

[d] Melissen: Packing 16, 17 or 18 circles in an equilateral triangle. Discrete Mathematics (1995)

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Circle packing (3)

Random space filling[e]**:** Attempts to solve the problem by iterative filling.

Problems

- Increasing inefficiency as more and more circles are packed.
- Situation worsens in case of region with an arbitrary shape—a usual outcome of natural object segmentation.

[e] http://paulbourke.net/texture colour/randomtile.

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Circle packing

Our solution

- Compute *Euclidean distance transform (EDT)*
- Use EDT to pack *denomination circles* in different segments.
- *DP technique* of solving the coin denomination problem.[f]
- *Recompute the EDT* of a segment after packing a circle and then greedily check feasibility of placing the next circle.

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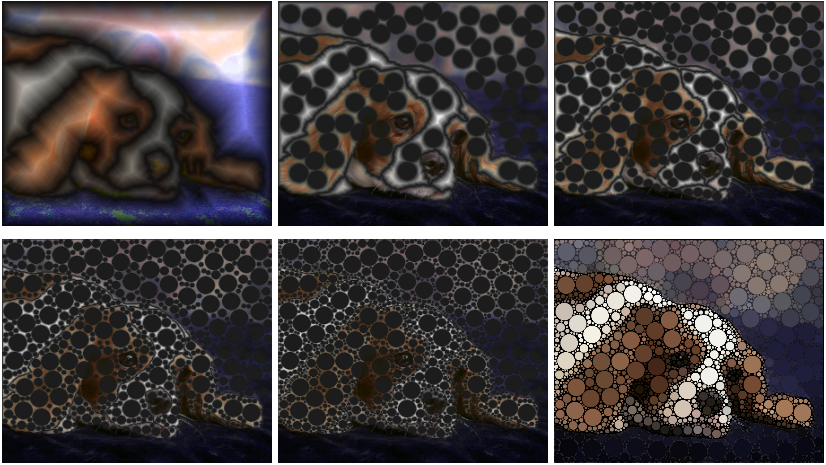
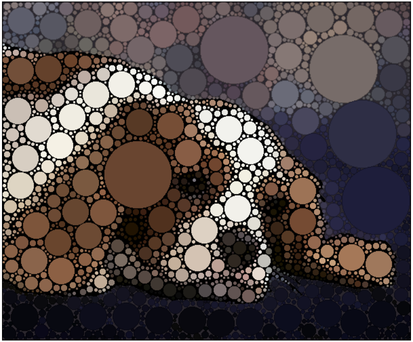
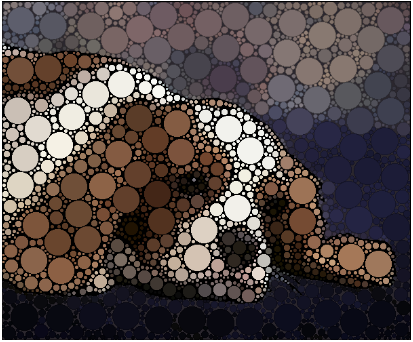


denomination set $D_2 = \{20, 10, 5, 3, 2\}$

Circle packing



$D_1 = \{\mathbf{200}, \mathbf{100}, \mathbf{50}, 20, 10, 5, 3, 2\}$



$D_3 = \{5, 3, 2\}$

Runtime

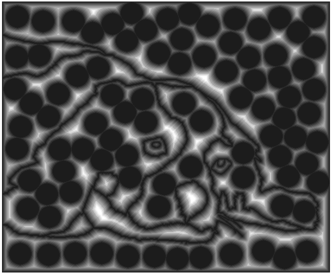
EDT → $O(n)$ *time*, $n = \#$ image pixels.

For packing circles of radius r

Local maxima finding
→ $O(n)$ *time.*

EDT recomputation in R after packing each circle
→ $O(|R|)$ time in *worst case.*

Let $c_R = \#$circles packed in R.
$\pi r^2 c_R \leqslant |R| \Rightarrow c_R = O(|R|/r^2)$.



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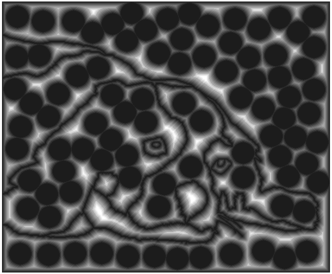

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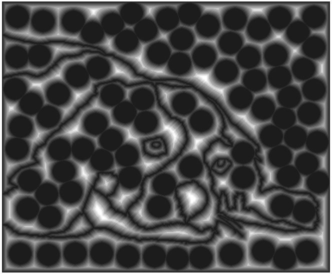

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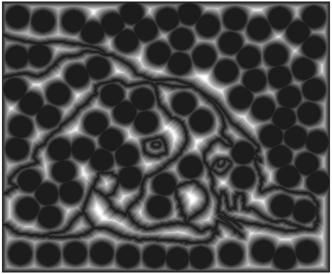

Runtime (2)

Thus, to compute EDT over all the r-radius circles packed in R, worst-case time complexity is

$$\begin{aligned} T(r) &= \sum_{R \subset I} c_R |R|, \text{ where } c_R = O(|R|/r^2) \\ &= \sum_{R \subset I} O\left(\left(\frac{|R|}{r} \right)^2 \right) \\ &= O\left(\sum_{R \subset I} \left(\frac{|R|}{r} \right)^2 \right) \\ &= O\left(\frac{n^2}{r^2} \right). \end{aligned}$$

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Runtime

(3)

If $|D| = k$, then total circle packing runtime is

$$T = \sum_{r \in D} T(r) = O\left(\frac{kn^2}{r_k^2}\right),$$

where $r_k = \min\{r_i : 1 \leqslant i \leqslant r_k\}$.

Note: We've not considered the decreasing nature of the effective packing area of a region R as circles are packed into it. So, $O(|R|)$ runtime for every recomputation of EDT is quite a loose bound.
A probabilistic analysis can possibly bring down the average time complexity.

Actual CPU time: Runtime increases with increase in

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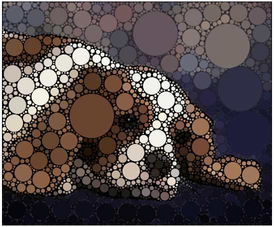
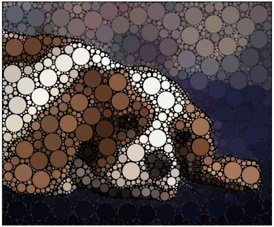


$|D_1| = 8$
69 secs.
$\rho = 0.827$

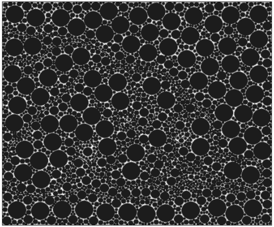


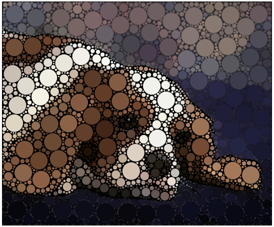
$|D_2| = 5$
18 secs.
$\rho = 0.824$

$|D_3| = 3$
16 secs.
$\rho = 0.749$

Color Rendition

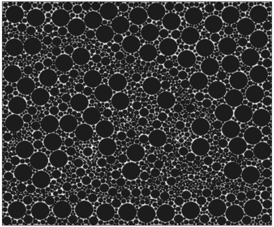
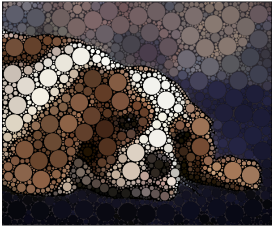
(1)





- Choose the primer colors for foreground and background.
- Color each circle by its average color from the original image.

Color Rendition

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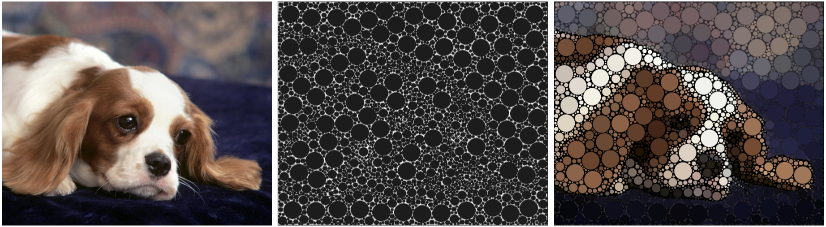

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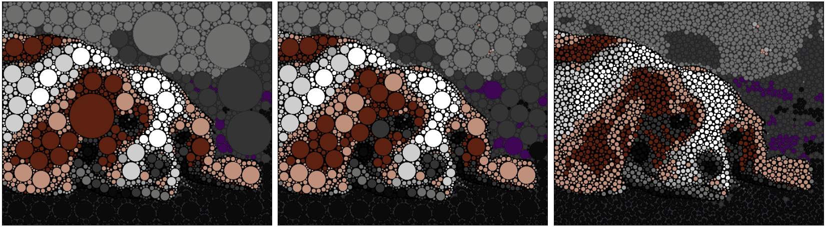
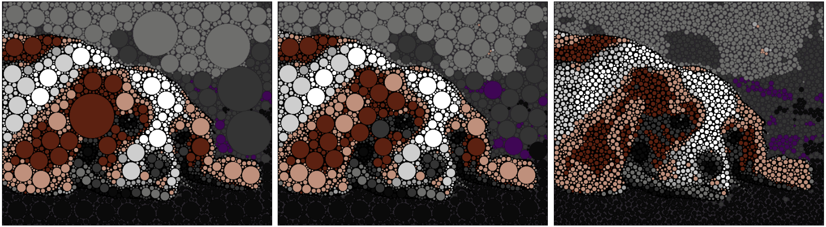


Macbeth Color Mapping

- Replace the circle color by the nearest color of the Macbeth chart.
- Nearness can be measured in different color subspaces (RGB, CIELUV, ...).

Color Rendition



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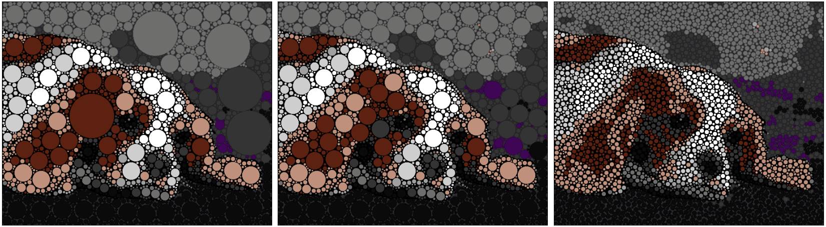

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Color Rendition

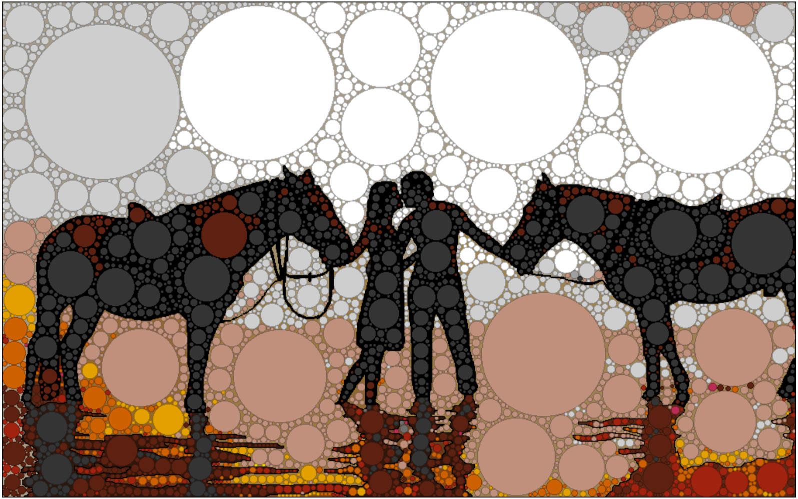
Color Rendition

(3)

No mapping

$\rho = 0.904$, *2 mins. 14 secs.*

Color Rendition

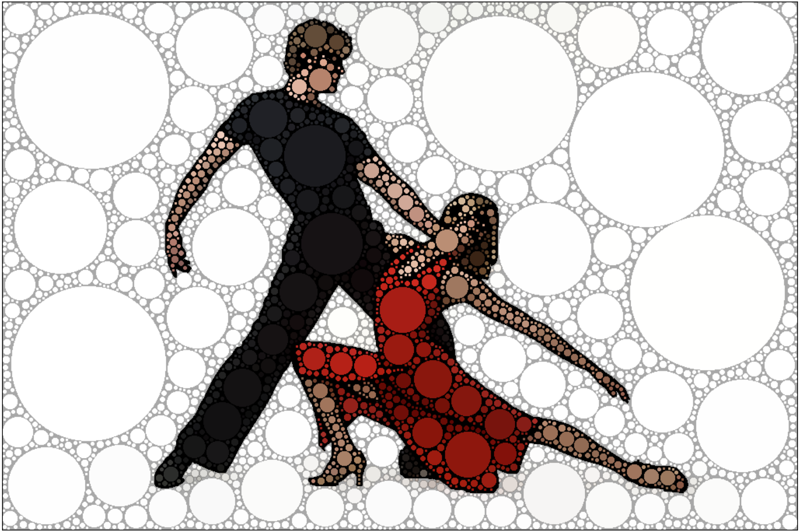


Macbeth color mapping to nearest

Color Rendition

Macbeth color mapping to 2nd nearest

Artwork Results



Artwork Results

Artwork Results

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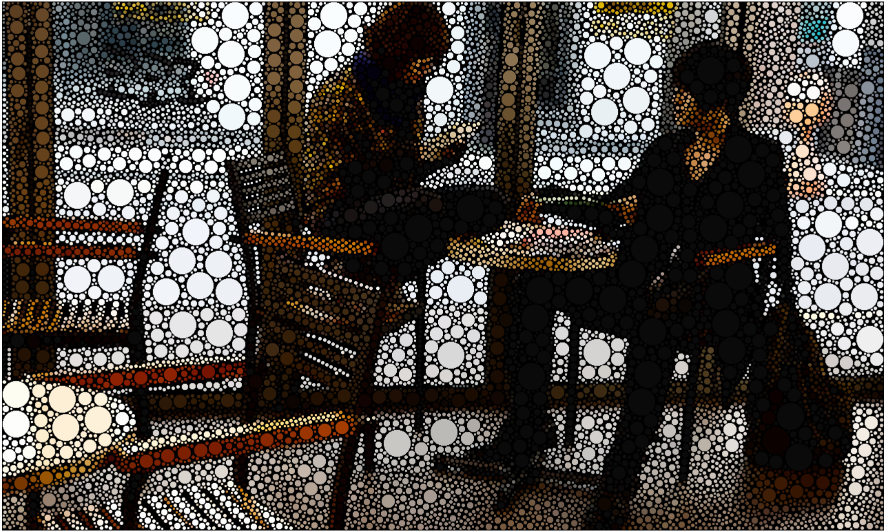

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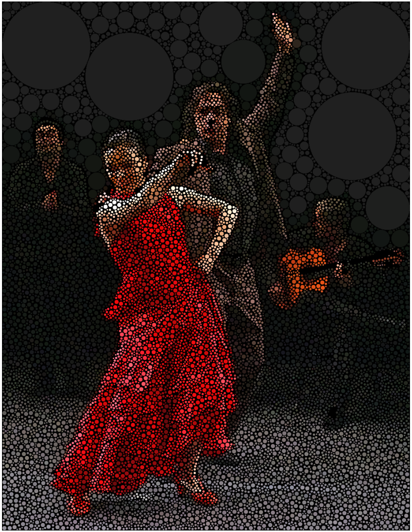

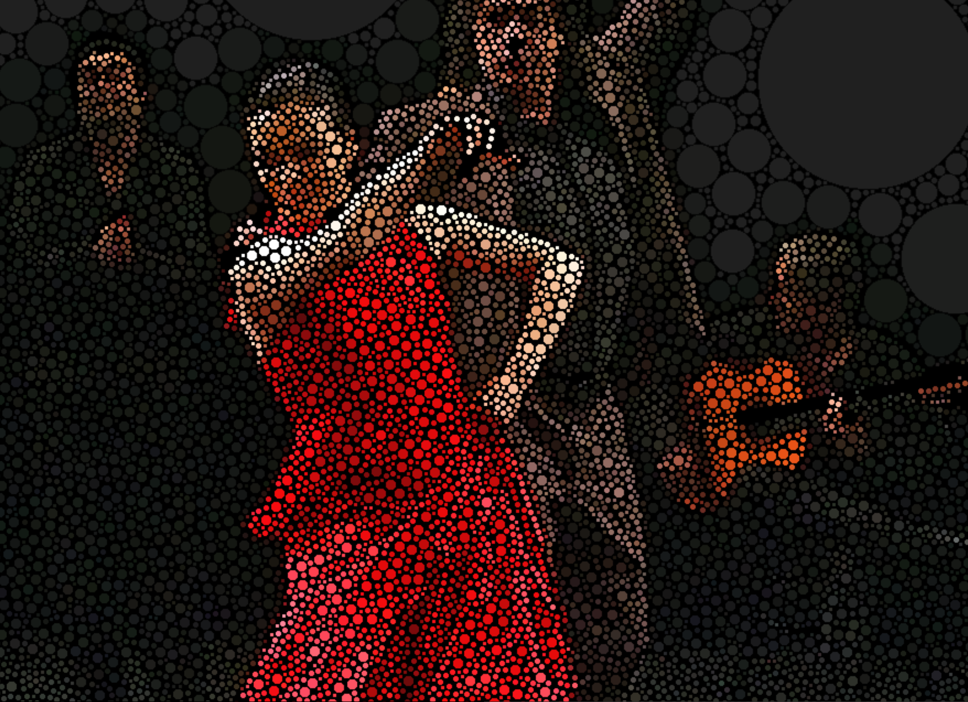


Thank You