Completeness Analysis of a Sanskrit Reader

Pawan Goyal and Gérard Huet

INRIA Paris-Rocquencourt, BP 105, 78153 Le Chesnay Cedex, France

Abstract. We analyse in this paper differences of linguistic treatment of Sanskrit in the Sanskrit Heritage platform¹ and in the Pāninian grammatical tradition.

1 General Methodology

The general assumption behind the design of the Heritage Sanskrit Reader is that sentences from classical Sanskrit may be generated as the image by a regular relation R of the Kleene closure W^* of a regular set W of words over a finite alphabet Σ . Think of W as the vocabulary of (inflected) words (padas) and of R as sandhi.

The computerized lexer underlying the Heritage Reader essentially proceeds by inverting relation R over the candidate sentence w in order to produce a finite sequence $w_1, w_2, ..., w_n$ of word forms, together with a proof that $w \in R(w_1 \cdot w_2 \cdot ... \cdot w_n)$. The word forms w_i must be justified being valid word forms of Sanskrit (i.e. padas), and some justification must be offered that the combination of such word forms makes sense. The first justification consists in exhibiting w_i as the lemmatization of some root stem, according to valid rules of morphology. The second justification consists in giving some dependency analysis of sentence w using assignments of semantic roles for the individual w_i 's consistent with their morphological analysis. Both kinds of justifications must be ultimately related to the traditional methods of Sanskrit grammar $(vy\bar{a}karana)$. That is, that each w_i corresponds to some w_i' , obtainable by a valid Pāṇinian derivation sequence, and that the concatenation of the sequence

¹ http://sanskrit.inria.fr

 $w'_1, w'_2, ..., w'_n$ yields by some valid Pāṇinian derivation a final sequence of phonemes w' equivalent in a strong sense to the original w.

Thus, to fix ideas with a concrete trivial example, the sentence (in Roman transliteration): $r\bar{a}mogr\bar{a}mamgacchati$, equivalently represented as: $r\bar{a}mogr\bar{a}mangacchati^2$, may be analysed as the sequence:

```
[r\bar{a}mah \ \{ \text{ nom. sg. m. } \}[r\bar{a}ma]\langle ah|g \rightarrow og \rangle]

[gr\bar{a}mam \ \{ \text{ acc. sg. m. } \}[gr\bar{a}ma]\langle m|g \rightarrow hg \rangle]

[gacchati \ \{ \text{ pr. } [1]ac. sg. 3 \}[gam]\langle \rangle]
```

where the various forms are lemmatizations of the basic stems $(pr\bar{a}tipadikas)\ r\bar{a}ma$, $gr\bar{a}ma$ and the root gam, respectively. The root gam (to go) being transitive (sakarmaka) and its form gacchati being in the active voice, we may assign the agent role (kartr) to the nominative form $r\bar{a}mah$ and the goal role (karman) to the accusative form $gr\bar{a}mam$, yielding the interpretation "Rāma goes to the village". It should then be a routine exercise (for a $vy\bar{a}karana$ specialist) to use the sequence of lemmas and phonetic rewrite rules as a guide to the generation of a Pāṇinian derivation of the original sentence.

The method looks hardly original, in view of modern computational linguistics, and it could be applied $mutatis\ mutandis$ to many natural languages, with Σ representing the set of phonemes of the language, W being the vocabulary of (inflected) words and R being some phonetic smoothing (called sandhi in Sanskrit). Often, at least in Western languages such as English or French, the phonetic treatment is given separately as a layer of speech understanding, words are strings over an alphabet of written characters, isolated by blanks in the written sentence, and more importance is assigned to a structural notion of syntax where word order is important. The basic assumption is that there is a clear hierar-

² The normalization procedure implemented in the Heritage Reader, where the $anusv\bar{a}ra$ before a consonant is replaced by the homogeneous nasal of the consonant, corresponds to the following rule in Pāṇini's grammar:

^{8.4.58} anusvārasya yayi parasavarņa \dot{n} : In sa \dot{m} hit \bar{a} , if \dot{m} is followed by a consonant in pratyāhāra yay, it is replaced by the homophonic nasal of the consonant.

chy of levels of linguistic description (traditionally called phonetics, phonemics, morphology, syntax, semantics, pragmatics), corresponding to more or less modular computational processes, and that the interfaces between these processes are the essential components allowing generation in one direction, and analysis in the reverse direction in a general model of universal linguistics. It could thus appear that the treatment of Sanskrit is just slightly biased, in having a significant sandhi component because of the faithfulness of its written representation to oral enunciation, and in having a shallow syntactic component because of the richness of its morphology. Indeed, the treatment of Sanskrit linguistics by Paul Kiparsky (2009) is essentially a precise elaboration of such processes, at least in the synthesis direction. One is thus hardly surprised at the general presentation of the methodology.

However, things are not that simple. For one thing, if this methodology was straightforward, one wonders why the traditional methods of linguistics analysis (vyākarana) do not explain sentence formation in this simple, intuitive, modular fashion. Indeed, the traditional Pāninian presentation of Sanskrit grammar by the trinity of the three sages (trimuni) of ancient India is an extremely complex interweaving of formal transformations operating at all levels in parallel, and it has been copiously criticized by linguists of the nineteenth century such as Whitney. However, there exists to this date no grammatical description of Sanskrit that is as precise and as complete as the traditional one, and Sanskrit grammar experts well aware of modern linguistics theories such as George Cardona and Peter Scharf (2009) maintain that mixing of levels is unavoidable for accounting for the complexity of Sanskrit grammar. Thus it remains a challenge to provide a modular description of computational linguistics processes adequate to recognize correct Sanskrit sentences, and there is to this date no better justification of such than to relate their operations to that of the tradition. We shall attempt to meet this challenge in the present paper, and to provide a rationale for the completeness of the Heritage Reader with respect to a well-identified subset of classical Sanskrit.

2 Morphology

2.1 Limiting the Recursion

The first difficulty we are facing is in the definition of the set W of words (padas). It cannot simply be explained as the set of inflected forms of a finite set of "atomic" words. For instance, it is possible to form compounds of arbitrary length, making the set of substantive bases ($pr\bar{a}tipadikas$) infinite. Furthermore, the process of turning a substantive base into a denominative verb is productive,³ even for compounds, although this last possibility is infrequent. If we examine the diagram given in (Kulkarni and Shukl, 2009), which purports to explain the recursive structure of Sanskrit morphology as authorized by the traditional grammar, it seems hopeless to invert it directly in order to obtain a complete parser. By contrast, the (simplified) finite-state diagram underlying the Heritage transducer given in Fig.1 reduces drastically the allowed recursions. For one thing, it forbids denominative verbal forms, except for the conjugated forms of a finite set of lexicalized verbal stems. Also,

- kyac: 3.1.8 supaḥ ātmanaḥ kyac. E.g. ātmanaḥ putram icchati
 →putrīyati.
- $k\bar{a}myac$: 3.1.9 $k\bar{a}myac$ ca. E.g. $\bar{a}tmana\dot{p}$ putram icchati → putrak $\bar{a}myati$.
- kyan: 3.1.11 kartuḥ kyan salopaḥ ca. E.g. śyena iva ācarata → śyenāyate.
- -kyaş: 3.1.13 lohitādiḍājbhyaḥ kyaş. E.g. alohito lohito bhavati →lohitāyate.
- nin: 3.1.20 pucchabhāṇḍacīvarāt ṇin. E.g. puccham udasyati
 →utpucchayate.
- nic: 3.1.21 mundamiśraślakṣṇalavaṇavratavastrahalakalakṛtatūstebhyaḥ ṇic. E.g. miśraṃ karoti →miśrayati.

Suffix nin is specific only to three nominal bases, kyas is specific to the $lohit\bar{a}digana$, while the other suffixes are applicable to any nominal base. Among these suffixes, the suffix nic is the most productive and there is a $ganas\bar{u}tra$: tatkaroti $tad\bar{a}caste$, which specifies this suffix after any nominal base in the sense of 'he does that', 'he behaves like', 'he says that'.

³ As per $P\bar{a}nini$'s grammar, the main $s\bar{u}tras$ corresponding to the nominal verbs ordain six different suffixes:

it makes many assumptions on compounding which must be explained and justified.

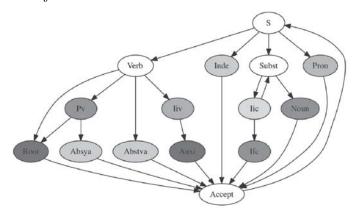


Fig. 1: A simplified lexical analyser

Let us explain briefly Fig.1, in order to relate it to the traditional terminology. Each colored oval, called a phase of the underlying automaton, corresponds to a finite set of forms. Going from top (node S) to bottom (node Accept) consists in following the arcs, selecting forms from the intervening phases, and glueing them with sandhi, in order to obtain one word (pada). The subdiagram below the Subst node builds substantival forms (subantas). The subdiagram below the Verb node builds verbal forms (tinantas). The Inde node builds indeclinable forms, the Pron node builds pronominal forms. Phase Noun contains declined forms of autonomous atomic substantive and adjective stems, found in the lexicon with their attested gender parameters. Phase Ifc contains the non-autonomous ones, usable only as the right-hand component of a compound, such as root substantival forms, -gam (gone to). Phase Iic contains bare stems of nouns (prātipadikas) such as deva-. Phase Root contains conjugated forms of roots, phase Pv contains attested sequences of preverbs (upasargas) such as \bar{a} , sam, $nir\bar{a}$, and $samabhivy\bar{a}$. Phase Auxi contains conjugated forms of the three auxiliary roots $bh\bar{u}$, as and kr, phase Iiv contains periphrastic forms of a substantive formed with an $-\bar{\imath}$ suffix (technically designated as cvi in Pāninian terminology). The path Iiv-Auxi allows generation of compound verbal forms such as pavitrīkaroti (he purifies). Phase Inde contains a mix of indeclinable forms, such as grammatical words (ca, $v\bar{a}$, api, etc.) and adverbs $(yath\bar{a})$. Absolutives in $-tv\bar{a}$ $(krtv\bar{a})$ are stored in Abstva, whereas absolutives in -ya, necessarily prefixed with a preverb, are put in phase Absya. Infinitive forms (kartum), which may or may not be prefixed with a preverb, appear both in Inde and in Absya.

This diagram is a simplification of the actual automaton transition graph of the Sanskrit Heritage lexical analyser, that has numerous other phase databanks to account for constructions such as periphrastic perfect, but it suffices to give a general idea and start the discussion.

2.2 Morphology Generation

Each of the coloured nodes of the diagram of Fig.1 represents a databank of forms. These data banks are filled in during a series of preliminary passes. First the lexicon is analysed to gather stems and their morphological parameters, such as permitted genders of nominal stems, allowed present classes (gana) and attested preverbs for roots. In a second phase, more stem generation occurs for roots, accounting for the various tenses/moods $(lak\bar{a}ras)$, as well as absolutives and infinitives, but also participles (adjectival krdantas) in ten varieties. Finally, inflexional morphology paradigms derive the inflected forms according to the morphological parameters, some of which being read from the lexicon, some of which being defined in specific tables.

It is not simple to relate our paradigmatic derivations and the operations of Pāṇinian grammar. Some stem operations relate to morpho-phonetic operations well identified in the central rules $(s\bar{u}tras)$, such as taking the phonetic grades of quna or vrddhi. Some are dispersed in various parts of the grammar such as stem substitutions. Still others, such as retroflexion, occur in the final section $(trip\bar{a}d\bar{i})$ of the $Ast\bar{a}dhy\bar{a}y\bar{i}$, which comprises a list of rewrite rules which are applied iteratively at the end of the derivation process, in some kind of "phonetic smoothing" phase. Since the forms stored in our databanks are to be matched to the surface realization of the sentence, this phonetic smoothing must be applied at the time of generation of these forms. Some careful analysis must be done in order to understand the mutual interaction of the retroflexion rules and of the external sandhi rules (which are used in segmentation in the Heritage Reader). This analysis will be given in Section 4.4 below.

The main discrepancy between the Pāṇinian processes and our reader operations is in the order of rewritings. Often the $s\bar{u}tras$ relevant to a given operation are dispersed in different sections of the $Ast\bar{a}dhy\bar{a}y\bar{\imath}$, and thus it is hard to give the trace of the needed $s\bar{u}tras$. However, in many cases, one can recognize the $s\bar{u}tras$ operations from the computer program. A complete analysis, in the specific case of future passive participles in -ya (yat, kyap, nyat), is given in Appendix A.

2.3 Variations in Sandhi

One related difficulty concerns the precise definition of the *sandhi* relation labeling implicitly the arcs of the diagram. For most arcs, it is what Western linguists call "external *sandhi*". However, for the arcs issued from the Pv phase, it adds retroflexion rules, necessary to explain verbal forms of verbs (or participles) prefixed by preverb sequences. Furthermore, the forms stored in the various databanks (Noun, etc.) use such retroflexion rules in the formation of their stems and of their inflexions.

The correctness and completeness of our method involves thus a careful assessment of the mutual interaction (feeding, bleeding) between the rules of the morpho-phonetic processes of stem formation and inflection, the rules of external sandhi, and the final $trip\bar{a}d\bar{\imath}$ rules.

3 Specific Problems of Compounding

3.1 Pre-Compounds vs. Compounds

Another set of difficulties concerns compounding. First of all, what our analyser recognizes by using the loop going through phase lic in Fig. 1 is not strictly speaking a compound $(sam\bar{a}sa)$, but what we call a pre-compound a mere sequence of lic bare stems ended in an inflected substantival form. Several possible compounds may be mapped to a unique pre-compound, representing the common frontier of the corresponding binary trees. Thus the sequence $\bar{a}nandamayakośah$ will parse as a unique pre-compound $\bar{a}nandamayakośah$, which has two possible interpretations as compositions of binary compounding, one of which is the intended one

 $(\bar{a}nanda < maya) < kośa$ in Gillon's notation (2009). This has advantages (a unique pre-compound parse instead of C_n parses of an n+1-component compound, where C_n , the n^{t_h} Catalan number, is an exponential function of n, as explained in (Huet, 2009), and drawbacks (some ulterior computation will be needed in order to resolve the ambiguity if one wants to get a handle on its meaning). Only at this ulterior stage one will be able to relate the precise compound construction to the relevant $s\bar{u}tras$.

3.2 Forms Allowed as Left Components of Compounds

Heritage Treatment

Another problem is that the left component of a compound is not always a bare stem - it may be a gendered stem (e.g. $durg\bar{a}p\bar{u}j\bar{a}$). It may even be a fully inflected form, like in so-called aluk compounds such as $gav\bar{a}mayana$ ("cows' travel" or year), where $gav\bar{a}m$ is the genitive plural of go. We consider this last construction as non-productive and assume that the finite set of such attested frozen forms is lexicalized. Other possible left components of compounds are particles which are not bare stems of stand-alone substantives, such as su, vi, dur. They must be accommodated in the Iic bank.

$A s t \bar{a} dh y \bar{a} y \bar{\imath}$ Treatment

The left component of a compound may undergo some operations as enumerated in $P\bar{a}nini$'s grammar and, therefore, is not always a bare stem. First, we have the problem of *aluk* compounds, where Iic can be a fully inflected form. The rules corresponding to *aluk* compounds are discussed in $P\bar{a}nini$ from 6.3.1 to 6.3.24. For example:

- 6.3.4 manasaḥ saṁjñāyām: aluk applies to a $tr\bar{t}\bar{y}\bar{a}$, which occurs after manas, when a constituent in combination follows and the derivate denotes a $saṁjñ\bar{a}$. Thus, $manas\bar{a}datt\bar{a}$. The Iic $manas\bar{a}$ is the inflected form of manas in instrumental case $(tr\bar{t}\bar{y}\bar{a})$.
- 6.3.7 $vaiy\bar{a}karan\bar{a}khy\bar{a}y\bar{a}m$ $caturthy\bar{a}h$: aluk applies to a dative form $(caturth\bar{\imath})$, which occurs after $\bar{a}tman$, when a constituent in combination follows and the derivate denotes a

name assigned by the grammarians. Thus, $\bar{a}tmanepadam$. The Iic $\bar{a}tmane$ is the inflected form of $\bar{a}tman$ in $caturth\bar{i}$.

Similar to the rules as enumerated above, the rules for *aluk* compounds are very specific and generate a finite set of compounds. Thus, this formation is non-productive and as a design decision in the Heritage system, we consider these compounds as frozen expressions and the finite set of these forms is lexicalized.

Another problem is that in some cases, the Iic stems may undergo some changes and may not be the same as the bare stem. These cases correspond to $P\bar{a}nini$'s rules 6.3.25 to 6.3.139. For example:

6.3.52 $p\bar{a}dasya$ $pad\bar{a}jy\bar{a}tigopahatesu$: The word $p\bar{a}da$ is replaced by pada, when $\bar{a}ji$, $\bar{a}ti$, ga or upahata combine to follow. Thus padopahatah, padagah.

6.3.70 $k\bar{a}re\ saty\bar{a}gadasya$: Augment mum is introduced to satya and agada, when $k\bar{a}ra$ combines to follow. Thus $satya\dot{n}-k\bar{a}rah$.

Similar to these rules, the rules in this section ordain various changes in the Iic of a compound. While most of the rules in this section are non-productive and the cases like 6.3.52 are being taken care of by declaring additional Iic stem pada for the word $p\bar{a}da$, some of the rules which are productive and may require special treatment are:

6.3.34 striyāh pumvat bhāṣitapumskādanūn samānādhikaraṇe striyām apūraṇīpriyādiṣu: In the place of a feminine word, the corresponding masculine form is substituted, when the feminine word has an actual corresponding masculine and does not end in the suffix $\bar{u}n$, if followed by another feminine word, but not when this word ends in a $p\bar{u}raṇa$ suffix or belongs to the list headed by $priy\bar{a}$. Thus, $darśan\bar{v}abh\bar{a}ry\bar{a}$ and not $darśan\bar{v}abh\bar{a}ry\bar{a}$.

6.3.61 ikah hrasvah anyah $g\bar{a}lavasya$: The final of a nominal which ends in a vowel belonging to the $praty\bar{a}h\bar{a}ra$ ik, with the exception of $n\bar{i}$, is optionally replaced by its short counterpart in the opinion of $G\bar{a}lava$. Thus, $gr\bar{a}man\bar{i}putrah$ and $gr\bar{a}man\bar{i}putrah$.

A careful assessment of the rule 6.3.34 reveals that we actually take care of the words generated using this rule without any special treatment. The Iic stems which may qualify for this rule are those feminine stems, which have been derived from a masculine stem and as a result of this rule application, these stems will be substituted by their masculine stems. In the Heritage system, the corresponding masculine forms will always be stored in the Iic banks and thus, analysing these compounds will not raise a special problem.

The case with the rule 6.3.61 is different because the Iic stem is shortened and may not be present in our Iic bank. However, this is an optional rule and some statistics from the corpora is required to figure out the productive nature of this optional shortening.

3.3 Forms Allowed as Right Components of Compounds Heritage Treatment

Still another problem concerns the forms allowed as right components of compounds. Sometimes, and this is the case of the important class of tatpurusa compounds, there is gender preservation of the right component. We are thus assured of finding the corresponding form in the Noun database, which contain forms of words consistent with their lexicalized gender. But this is not true of the exocentric $(bahuvr\bar{\imath}hi)$ compounds, where a substantive may be lifted to an adjective used in a different gender, such as caturmukhaḥ "who has four heads", since mukhaḥ is not an autonomous form of neuter stem mukha. This forces us to add forms such as mukhah to the Ifc database. A similar problem arises with the so-called $avyay\bar{\imath}bh\bar{a}va$ compounds, where an adverb "turns to indeclinable" its right component, usually by coining a neuter-like form, like in yathāśraddham (according to your convictions), where the right component is similar to the accusative form of a hypothetical neuter stem śraddha induced from the originally lexicalized feminine stem $\acute{s}raddh\bar{a}$.

$Ast\bar{a}dhy\bar{a}y\bar{\imath}$ Treatment

Pāṇini's $s\bar{u}tra~2.2.24~(anekam~anyapad\bar{a}rthe)$ states that many syntactically related padas combine in a $bahuvr\bar{\imath}hi$ compound and the compound denotes the meaning of something other than its own

constituents. Therefore, it acts as an adjective and, like other adjectives in Sanskrit, can take any gender.

In the case of $avyay\bar{\imath}bh\bar{a}va$ compounds, the Ifc is changed to neuter (shortened)⁴ and the compound gets the $sa\dot{m}j\tilde{n}\bar{a}$ 'avyaya'⁵ and, therefore, the nominal case ending is deleted.⁶ Some exceptions where the nominal case ending is not deleted are stated in the following rule:

2.4.83 na avyayībhāvāt ataḥ am tu apañcamyāḥ: A case ending occurring after an avyayībhāva compound is not deleted when the compound ends in a, instead the case ending is replaced by am except when it is in ablative $(pañcam\bar{\imath})$.

The generation of $avyay\bar{\imath}bh\bar{a}va$ compounds can be implemented by adding two phases. The first phase corresponds to the possible left-hand sides which can be listed as a finite set of words, corresponding to the $s\bar{u}tras$ 2.1.5 to 2.1.19, such as $\{adhi, pari, anu, antar, upa, su, dur, nir, ati, iti, tat, prati, yath\bar{a}, sa, y\bar{u}vat, apa, bahir, pr\bar{u}k, \bar{a}, abhi, p\bar{u}re, madhye\}$ and the set of numbers. The second phase corresponds to the forms of the second component, which must be constructed from ordinary $pr\bar{u}tipadika$ by various changes, as described in the grammar. If the $pr\bar{u}tipadika$ ends in a long vowel, it is changed to the corresponding short vowel (neuter stem). Some other stem changes are described in the rules 5.4.107 to 5.4.112. For example:

5.4.107 avyay $\bar{\imath}bh\bar{a}ve$ śaratprabhrtibhya \dot{n} : For the pr \bar{a} tipadikas listed in the śarad \bar{a} diga $\dot{n}a$, the suffix $\dot{\imath}ac$ is added. Thus, śarad can be stored as śaradam, obtained by the following sequence:

 $\acute{s}arad + \acute{t}ac \xrightarrow{5.4.107} \acute{s}arada \xrightarrow{2.4.83} \acute{s}aradam.$

5.4.111 jhayaḥ: For the $pr\bar{a}tipadikas$ ending in a consonant belonging to $praty\bar{a}h\bar{a}ra$ jhay (the first four letters of the sound classes), a suffix tac (a) is optionally added. Thus, the right hand side corresponding to sarit can be stored as two forms $\{sarit, saritam\}$, where saritam is obtained by the same process as described above for saradam.

 $^{^4}$ 2.4.18 avyayībhāvah ca and 1.2.47 hrasvah napumsake prātipadikasya

 $^{^5}$ 2.1.5 $avyay\bar{\imath}bh\bar{a}va\dot{h}$

 $^{^{6}}$ 2.4.82 $avyay\bar{a}t$ $\bar{a}psupah$

In effect, the forms of the second component correspond to the following two-stage method:

- 1. Stem changes from long to short and other changes as prescribed in the rules 5.4.107 to 5.4.112.
- 2. If the stem obtained from the first step ends in a, add the suffix am to it.

3.4 Intra-Compound Sandhi

Finally, we have to make sure that the components of a compound are glued together according to the sandhi rules that are assumed by our transition transducer. In general, external sandhi is used. However, it may happen that retroflexion occurs, as in $r\bar{a}m\bar{a}yana$, which is therefore not obtainable by external sandhi from its components $r\bar{a}ma$ and ayana. We shall assume that such frozen forms, usually occurring only in proper names, are not the result of a productive process, and can be obtained by proper lexicalization. But another class of compounds, called $pr\bar{a}di$ compounds, take retroflexion regularly. This is specially frequent with participles prefixed with preverbs, but also with other primary substantival formations (krdantas) from roots. In order to deal with those, we have to introduce new databanks of participial forms, accessible from yet another copy of preverb sequences, with retroflexion allowed on the corresponding transition, as shown in the more complete state diagram of Fig. 2. This extension demanded an extension of our morphological generator, which had to provide derivational morphology and not just flexional morphology. The $ast\bar{a}dhy\bar{a}y\bar{i}$ treatment of the retroflexion across word and compound boundaries has been dealt with exclusively in Section 4.4.

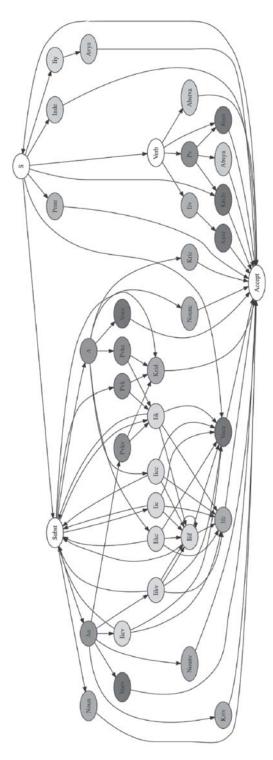


Fig. 2: A more complete lexical analyser.

Such intricate problems of compounding have already been discussed at length in other papers. We shall concentrate in the following on the justification that the morpho-phonetic devices used in our parser are indeed consistent, from both points of view of correctness and of completeness, with the rules defined in the $A s t \bar{a} - dhy \bar{a}y \bar{t}$, specially in view of their different order of application.

4 Completeness of the Segmenter

4.1 Basic Zen Technology

The Heritage platform is based on a functional toolkit for computational linguistics called Zen (Huet, 2002). This technology builds on a central applicative data structure of tries or lexical trees. Tries are a commonly used data structure for representing lexicons. One may see a trie as a deterministic finite state automaton recognizing the lexicon as a regular set of words. The maximal sharing of this data structure obtained by representing the underlying tree as a dag (directed acyclic graph) yields the corresponding minimal automaton. The Zen toolkit generalizes this observation to the case of general (i.e. possibly infinite) regular sets by representing the control graph of finite state machines such as finite automata and transducers as an annotated trie, amenable to optimum sharing as well. Furthermore, automata and transducers are seen as specific instantiations of very general relational abstract machines, called effective Eilenberg machines, admitting uniform simulators (Huet and Razet, 2006).

The first real-scale application of this Zen technology was to show how to solve efficiently the inverse problem of external *sandhi* (Huet, 2005), represented as a *juncture rewrite system*. Let us recall briefly the key notions.

We model external sandhi with rewrite rules of the form $u|v \to w$, where u, v and w are words (standing for strings of phonemes). Such a rule represents the rational relation that holds between all pairs of strings (from now on we use strings and words interchangeably) $\lambda u|v\rho$ and $\lambda w\rho$, for λ and ρ any strings. The symbol | stands for word juncture. The sandhi problem may then be posed as a regular expression problem, namely the correspondence between $(L \cdot |)^*$ and Σ^* by relation \mathcal{R} , where Σ is the word alphabet (our

extended phoneme alphabet), L is the set of inflected forms generated from the lexicon, and \mathcal{R} is the rational relation that is the concatenation closure of the union of the rational relations corresponding to the sandhi rules. This presentation is a standard one since the classic work of Kaplan and Kay (1994).

Note that the sandhi problem is expressed in a symmetric way. Going from $z_1|z_2|...,z_n| \in (L \cdot |)^*$ to $s \in \Sigma^*$ is generating a correct enunciation s with word forms $z_1, z_2, ... z_n$, using the sandhi transformations. Whereas going the other way means analysing the sentence s as a possible phonemic stream using words from the lexicon as amalgamated by sandhi. It is this second problem (sandhi viccheda) we are interested in solving, since sandhi, while basically deterministic in generation, is strongly ambiguous in analysis.

Huet (2005) explains how to use the Zen toolkit to perform this analysis automatically. It then goes on to give a formal justification of the methodology, under two assumptions, called strictness and non-overlapping. In a nutshell, the algorithm is sound (all outputs are plausible solutions in the sense of yielding the input sentence by sandhi), complete (all such solutions are enumerated), and terminating (the set of solutions is finite and the enumeration algorithm always terminates). Let us now examine the correctness assumptions. The strictness condition is simply that in every sandhi rule $u|v \rightarrow w$ the right-hand side word w is not empty and this is trivially satisfied. The non-overlapping condition involves both the lexicon L component and the rewrite rule R component. It expresses that there can be overlap of juncture rules of R within one word of L. We say that there is overlapping of rules $u|v\to w$ and $u'|v' \to w'$ within word $z \in L$ whenever the right-hand side v of the first rule, initial substring of z, overlaps with the terminal substring of z identical with the left-hand side u' of the second rule: |z| < |v| + |u'|. In other words, the first rule potentially bleeds the second one (in the sense of blocking its application). Since rules of external sandhi are very local (|v| = 1, and |u| = 1 or u = ah or u $= \bar{a}h$), such overlapping can occur only for very short words in L, of length at most 2. Furthermore, since neither ah nor $\bar{a}h$ are legal autonomous forms, this leaves us to consider only the rare words of length 1. There are basically two such words to consider in classical Sanskrit, besides the Vedic exclamation u which is not used in an autonomous manner in classical Sanskrit, but only in combination with other particles, marking a hiatus situation not amenable to sandhi (pragrhya). These two words are the preverb/preposition \bar{a} , and the negation prefix a. We shall deal with these two monophonemic words in the following two sections.

4.2 Phantom Phonemes

We are dealing in this section with sandhi interference in the case of the \bar{a} preverb. Let us start with an example, the simple imperative sentence "come here". The sentence is built up from the word iha (here) and the imperative form ihi of root i (to go), prefixed by preverb \bar{a} . If we build the imperative form *ehi* (come), and consider the sequence iha ehi, we would obtain by sandhi the wrong *ihaihi. The mistake comes from a too hasty glueing of the preverb to the root form, in a situation where sandhi is indeed not associative, precisely because of overlapping. We should first glue iha and \bar{a} , getting $ih\bar{a}$. Now glueing $ih\bar{a}$ and ihi by sandhi leads to the correct sentence *ihehi*. This example shows that the terminology "preverb" is actually misleading. Preverbs are just particles which have shifted from the Vedic language to the Classic one from a role of postposition to a preferred position of prefix to root and primary derivatives (krdantas). But they have kept enough autonomy as individual words to interact with their predecessor, in left-toright phonetic interaction, before the resulting utterance interacts with the root form. This vindicates our choice not to generate precooked forms obtained by glueing preverbs to root forms, but rather to consider (sequences of) preverbs as an autonomous phase in lexical analysis. This has the added interest to keep only one root form, instead of all the verb forms with preverbs attached.

 symbols, noted respectively *a, *e, *o and *r, for the four vowel varnas. Thus *a replaces a or \bar{a} , *e replaces i or $\bar{\imath}$, *o replaces u or \bar{u} , *r replaces \bar{r} or \bar{r} . The new rules of sandhi are all instances of $u|^*v \to w$, for u any vowel, *v the phantom of vowel v, w the result of sandhi of z and v, for z the result of sandhi of u and \bar{a} . For the above example, we add to the bank of root forms the special form *ehi, and we analyse ihehi as the glueing of iha with *ehi, using special sandhi rule $a|*e \to e$.

Correspondence with $Ast\bar{a}dhy\bar{a}y\bar{\imath}$ Treatment

In terms of $P\bar{a}n$ inian derivation, the concept of phantom phonemes can be shown to be in correspondence with the following rules:

6.1.85 antādivat ca: A replacement in the place of the preceding and following sound segments in $sa\dot{m}hit\bar{a}$ is treated as the final of what precedes and initial of what follows. 6.1.95 $om\bar{a}\dot{n}o\dot{h}$ ca: When a vowel a is followed by the \bar{a} of $\bar{a}\dot{n}$ or o of om, the second is a replacement in place of both.

So, when we reach at the stage iha + ehi in the Pāṇinian derivation, because of the metarule 6.1.85, the second word is as if starting with \bar{a} of $\bar{a}n$ and therefore, 6.1.95 finds its scope and replaces the a of iha and e of ehi by the second (which is e), giving the desired form ihehi. If we try to make a correspondence between the Pāṇinian derivation and phantom phonemes, we can think of these phantom phonemes as an indication that these have been obtained because of sandhi between \bar{a} and another vowel and, therefore, will follow the sandhi rule 6.1.95. In general, if we have a sandhi rule such that:

$$[\bar{a} + \text{Vowel x} = \text{Vowel y}],$$

we can replace the Vowel y with a phantom phoneme *y which follows the following *sandhi* rule after the vowel a:

$$[a + *y = y].$$

Replacing y by *y corresponds to the metarule 6.1.85, where the phonetic form is preserved but an indication is provided that it comes from the element \bar{a} , while the *sandhi* rule 'a + *y = y' corresponds to 6.1.95, where the two phonemes are replaced by the second.

4.3 Privative Compounds

Treatment of Privative Compounds in the Heritage Platform

In this section, we shall examine possible overlappings with the privative particle a. First of all, we emphasize that the morpheme a used for augment forms of the past tenses such as imperfect or aorist is not to be considered here, since it applies only in the generative morphology of root forms, which is assumed to be done in a previous phase (the construction of the Verb database of forms).⁷

It turns out that the privative particle does not lead to an interference situation, due to its dual nature; indeed, it is marked by the $pada\ a$ in front of consonants, but by the $pada\ an$ in front of vowels. Since a combines with a preceding phoneme either to form a syllable ended in a in case of a consonant, and to a vowel otherwise, it is clear that no sandhi overlap of the result may occur with the following $pr\bar{a}tipadika$. Note that the alliteration nasal n forbids the analysis of phoneme \bar{a} as an arbitrary succession of negations, should a alone be cumulative, leading to hopelessly ambiguous statements.

Thus it is enough to give the analysis of privative compounds by splitting the Noun database into two subsets Nounc (starting with a consonant) and Nounv (starting with a vowel), and by having distinct phases A or An, corresponding to unit databases representing the two respective strings a or an. The state space of the segmenter links phases A to Nounc and An to Nounv. A similar duplication is needed for the Iic phase, as well as for the Voc phase of vocative forms. This leads to a significant increase of the number of phases, as witnessed by Fig. 2, but no essential complication.

It should be emphasized, though, that the duplication of some phase does not lead to a duplication of the corresponding database. It is straightforward to share Noun, Nounc, and Nounv by only duplicating the first layer of the corresponding dags, and profiting of the fact that vowels are listed before consonants, leading to more

⁷ Indeed, if Sanskrit analysis was attempted without this initial generative phase, the inverting of augment would be a major hurdle, as well as retroflexion analysis; we do not believe such analysis, inverting Pāṇini's grammar rules fully down to root stems, to be computationally tractable.

sharing. There is thus very little increase of the space requirements of these databases, in good adequacy with the regime of economy $(l\bar{a}ghava)$ of the Zen machinery.

Treatment of Privative Compounds in the Aṣṭādhyāyī

The $na\tilde{n}$ compounds correspond to the following rules in Panini's grammar:

6.3.73 $nalopah na\tilde{n}ah$: In a compound, when $na\tilde{n}$ is followed by another word, the n of $na\tilde{n}$ is deleted.

6.3.74 $tasm\bar{a}t$ $nu\underline{t}$ aci: In a compound, if $na\tilde{n}$ is followed by a word starting with a vowel, the augment $nu\underline{t}$ is inserted after the n has been deleted.

Together, these two rules imply that $na\tilde{n}$ is replaced by a and an if the following word in the compound starts in a consonant and vowel respectively. In the Heritage system, a and an are denoted by two distinct phases A and An and the noun database is split into Nounc and Nounv for the nouns starting with a consonant and the nouns starting with a vowel, respectively. The phase A is linked to Nounc (6.3.73) and An is linked to Nounv (6.3.74).

4.4 Analysis of Retroflexion Across Word and Compound Boundaries

The retroflexion rules in Pāṇini's grammar appear in the $trip\bar{a}d\bar{\imath}$ section and as the other rules in this section, are applied as phonetic smoothing of the final forms. The retroflexion rules can be divided into two categories:⁸

Retroflexion rules for $n \rightarrow n$. Retroflexion rules for $s \rightarrow s$.

For a system that only uses word generation mechanism for handling sentences, handling retroflexion rules is not trivial. A parser based on an underlying word-generating device works on the principle that the word forms are stored and a compound as well as a pada

⁸ Please note that we will not discuss the *sandhi* rules such as 8.4.41 $stun\bar{a}$ stuh, which can also cause the changes from $s \to s$ and $n \to n$.

with external sandhi is analysed by glueing together the constituent word elements using external sandhi. These external sandhi rules are assumed by the transducer a priori and do not include retroflexion rules. So, while the constituent words themselves would have gone through retroflexion if the conditions are present within that word, it would not be the case if the condition for retroflexion is present in another word. Let us go back to the example $r\bar{a}m\bar{a}yana$. The two constituent words forming this compound are $r\bar{a}ma$ and ayana (ayana, the independent entry). A parser based on an underlying word generating device would have problems recognizing the term ayana because the independent word ayana does not have such retroflexion and the condition for retroflexion in ayana is triggered by the preceding segment $r\bar{a}ma$. We shall look into this issue by analysing the two cases of retroflexion separately.

Retroflexion Rules for $n \rightarrow n$

The main rules for this retroflexion in Pāṇini's grammar are the following:

8.4.1 $ras\bar{a}bhy\bar{a}m$ nah nah $sam\bar{a}napade$: A replacement in n comes in place of n when n occurs in close proximity, preceded by r or s in the same word.

8.4.2 $atkupv\bar{a}nnumvyav\bar{a}ye$ api: The above replacement takes place even if intervened by any phoneme in the set $\{at^9, ku^{10}, pu^{11}, \bar{a}n, num\}$.

The main rule 8.4.1 uses the term $sam\bar{a}napade$ (in the same word), which implies that this retroflexion can be applied in a single pada only. It clearly overrides the possibility of retroflexion in the case of external sandhi across word boundaries. Thus agnir + nayati = agnirnayati. This settles the problem of retroflexion in the case

 $[\]overline{g} \quad \underline{a} = Vowels \cup \{h, y, v, r\}.$

 $k\dot{u} = \{k, kh, g, gh, \dot{n}\}.$

 $pu = \{p, ph, b, bh, m\}.$

This example is interesting because of the fact that the generative model of $Ast\bar{a}dhy\bar{a}y\bar{\imath}$ uses the context from the complete sentence, in contrast with a generative model that first generates the separate final forms and, then, glues these together to generate the sentence. For this specific case of agnirnayati, the separate final forms in a word-

of external sandhi. However, Pāṇini lists many rules for retroflex-

generating device would be agnih and nayati. However, there is no rule in $trip\bar{a}d\bar{\iota}$ which can give the following desired form agnirnayati, starting with the input agnih + nayati, i.e.

```
agnih + nayati \rightarrow agnirnayati.
```

In this example, the final form stored by a word-generating device will not give the correct form in external sandhi. However, it is to be noted that the only instances, which can lead to this problem are the rules which have a specific condition of $avas\bar{a}na$. A pada in context of other words does not get this condition unless it is the final pada of a sentence. There are two rules in $trip\bar{a}d\bar{\imath}$ with the condition of $avas\bar{a}na$:

8.3.15 kharavasānayoḥ visarjanīyaḥ: The phoneme r is changed to h if followed by a phoneme in pratyāhāra khar or in avasāna. 8.4.56 $v\bar{a}$ avasāne: A phoneme belonging to the set jhal is changed to a phoneme in the set car optionally if the right context is avasāna.

Thus, the rule 8.4.56 changes $v\bar{a}g$ to the optional forms $v\bar{a}k$, $v\bar{a}g$ in $avas\bar{a}na$, which would be stored as the final forms in a word-generating device. It might have caused problem if the external sandhi between $v\bar{a}k + hari$ gives a different form that $v\bar{a}g + hari \xrightarrow{8.4.62} v\bar{a}gghari$. Thankfully, there is another rule 8.2.39 ($jhal\bar{a}m$ $ja\acute{s}ah$ ante), which can convert this final k back to g. Thus,

$$v\bar{a}k + hari \xrightarrow{8.2.39} v\bar{a}g + hari \xrightarrow{8.4.62} v\bar{a}gghari$$

gives the same form as $v\bar{a}g + hari$ and the sandhi rules $\{k + h = ggh, g + h = ggh\}$ will give the correct sandhi, in correspondence with $trip\bar{a}d\bar{\imath}$ rules.

We will now discuss the methodology adopted by the system to handle this situation created by the condition of $avas\bar{a}na$ in 8.3.15. The system stores the final form as to what it would be before applying that particular rule. So, the final form is stored as ending in r and the following external sandhi rules are applied:

```
r + khar \xrightarrow{8.3.15,8.3.34} s + khar.

r + \text{all phonemes } -khar \rightarrow \text{no change.}

r \text{ (end of } pada) \xrightarrow{8.3.15} h \text{ (end of } pada).
```

ion across constituent words in a compound formation, specifically rules 8.4.3 to 8.4.13. Let us consider rule 8.4.3 here:

8.4.3 $p\bar{u}rvapad\bar{a}t$ $sa\tilde{n}j\tilde{n}\bar{a}y\bar{a}m$ $aga\dot{p}$: A replacement \dot{p} comes in place of n, when n occurs in a pada, combined after another pada, containing r or \dot{s} as the condition of replacement, but not containing g, provided the derivate denotes a name.

This rule clearly gives the cases for retroflexion across constituent words (pada) in a compound $(sam\bar{a}sa)$. Now, questions have been raised in the tradition whether 8.4.3 is an operational rule or a restrictive rule (Sharma, 2003), that is, whether the retroflexion across pada was available from the rules 8.4.1 and 8.4.2. Let us describe these two views in brief below.

Operational view: A compound cannot be accepted as a single pada. Therefore, in dru-nasa, the retroflexion is not available by the previous rules and 8.4.3 prescribes this operation, making 8.4.3 to be an operational rule.

Restrictive view: Since nominal endings are introduced after a compound nominal stem to derive a single *pada*, the compound as a whole constitutes a single pada. Therefore, retroflexion in compounds was available from the previous rules. The rule 8.4.3 restricts this availability only to the compounds, denoting a name.

Whether we accept the operational view or the restrictive view, it leads to the same interpretation, i.e., in the case of compounds, retroflexion across pada occurs only in the case of $samj\tilde{n}\bar{a}$ (names) such as $r\bar{a}m\bar{a}yana$ and $s\bar{u}rpanakh\bar{a}$. It suggests that these compounds can actually be lexicalized as frozen expressions.

Apart from 8.4.3, other rules dealing with retroflexion across pada in a compound have a limited scope since the conditions for these rules are the specific lexical entries. For instance:

8.4.4 vanam puragāmiśrakāsidhrakāsārikākoṭarāgrebhyaḥ: When vana occurs in combination after r or s contained within a previous pada constituted by puragā, sidhrakā, sārikā, koṭarā or agre, the n of vana undergoes retroflexion.
8.4.8 vāhanam āhitāt: When $v\bar{a}hana$ occurs in combination after r or s contained within a previous pada constituted by a word signifying 'that which is carried', the n of $v\bar{a}hana$ undergoes retroflexion.

Similarly, other rules in this section also have a limited scope and these cases can be lexicalized. However, there is another rule in this section, which is general enough:

8.4.11 $pr\bar{a}tipadik\bar{a}ntanumvibhaktiṣu\ ca$: A replacement in \bar{n} comes optionally in place of n, which either occurs at the end of a nominal stem or occurs as a part of num or vibhakti, if preceded by a pada containing r or \bar{s} as the condition of replacement. For example, $m\bar{a}\bar{s}a-v\bar{a}pin+$ nominative dual $\to m\bar{a}\bar{s}av\bar{a}pinau$ (n occurs at the end of the nominal stem $v\bar{a}pin$) $m\bar{a}\bar{s}a-v\bar{a}pa+$ nominative plural $\to m\bar{a}\bar{s}av\bar{a}p\bar{a}ni$ (n of num) $m\bar{a}\bar{s}a-v\bar{a}pa+$ instrumental singular $\to m\bar{a}\bar{s}av\bar{a}pena$ (n of vibhakti).

Rule 8.4.11 is a general rule, not specific to the proper names only and, therefore, it is not clear whether lexicalizing such occurrences would suffice. This rule, however, is an optional rule and it needs to be verified that there are sufficient corpus occurrences of this specific case of retroflexion. To access the number of compounds which may fall in this category, that is, satisfy the conditions of 8.4.11, we gathered some statistics from the attested compounds in the Monier-Williams (MW) (1999) dictionary. The compounds satisfying the conditions for 8.4.11 can fall in the following two categories:

The compounds ending in n, where the left constituent of the compound may trigger the retroflexion in the right constituent: There are fifty-seven such compounds attested in MW. Example, $agra-g\bar{a}min$ might decline as $\{agra-g\bar{a}minau, agra-g\bar{a}minau\}$. The compounds where there is no n in the right constituent but the left constituent may trigger the retroflexion when the compound word declines: There are 509 such compounds, attested in MW. Example, $s\bar{a}ra-bh\bar{u}mi$ might decline as $s\bar{a}ra-bh\bar{u}min\bar{a}$, $s\bar{a}ra-bh\bar{u}min\bar{a}$.

We are now left with the question whether the optional retroflexed forms allowed by 8.4.11 actually appear in the corpus and if they do, how many such forms are there? An analysis into corpus statistics is needed for all these forms.¹³ As of now, we do not have

¹³ It is not possible to get these statistics from MW because this retroflexion occurs for the inflected forms.

any statistics regarding the occurrences of such forms with/without retroflexion in a corpus.

Retroflexion rules dealing with preverbs for the cases of $pr\bar{a}di$ compounds as well as in a single pada are listed from 8.4.15 to 8.4.35. The main $s\bar{u}tra$ 8.4.14 states:

8.4.14 $upasarg\bar{a}d$ $asam\bar{a}se$ api nopadeśasya: If a preverb (upasarga) contains the r or s as the condition of retroflexion, it can cause retroflexion in the following verb if the verb contains n in the initial citation (upadeśa). Thus $pran\bar{a}yakah$.

Retroflexion Rules for $s \rightarrow s$

The main rules in $P\bar{a}nini$'s grammar corresponding to this retroflexion case are the following:

8.3.55 apadāntasya $m\bar{u}rdhanya\dot{p}$: A replacement in $m\bar{u}rdhanya$ (retroflex) comes in place of that s, which does not occur at the end of a pada.

8.3.57 inkoh: A non-final s is replaced with s, when the same occurs after in^{14} or ku.

 $8.3.58 \ numvisarjan\bar{\imath}ya\acute{s}arvyav\bar{a}ye \ api$: The replacement takes place even if intervened by num, $visarjan\bar{\imath}ya$ or $\acute{s}ar$. ¹⁵

For this case of retroflexion, $P\bar{a}nini$ does not use the term $sam\bar{a}na-pade$. Therefore, we need to consider both the word and compound boundary cases in our analysis. However, this retroflexion has a different property. The number of intervening phonemes can at most be 1 and, therefore, it limits the cases where this retroflexion can occur across words. We will enumerate all the possibilities where the condition for this retroflexion may be obtained across the word boundaries. Before going into that, we define the following sets:

 C_s : The set of phonemes that trigger this retroflexion: (8.3.57: $i n \cup k u$).

 I_s : The set of phonemes that can intervene between any phoneme $p \in C_s$ and 's', which is to be retroflexed: (8.3.58: n, h, s, s, s).

 $[\]overline{\begin{array}{l} 14 \text{ } i \underline{n} = \{\text{Vowels - } \{a, \ \bar{a}\}\} \cup \{h, \ y, \ v, \ r, \ l\}. \\
15 \text{ } \acute{s} a r = \{\acute{s}, \ \emph{s}, \ s\}.$

Let us denote the left and right constituents under consideration by P_1 and P_2 , respectively. We are looking for the cases where a phoneme in P_1 can trigger retroflexion in P_2 . We will have the following cases:

Case 1: $p \in C_s$, $q \in I_s$ and P_1 ends in 'pq', P_2 starts with an s.

8.3.111 $s\bar{a}tpad\bar{a}dyo\dot{h}$ states that if the s occurs in the beginning of a word, it will not undergo retroflexion. In this case, s occurs in the beginning of P_2 and, therefore, this case is overruled by the grammar.

Case 2: $p \in C_s$, P_1 ends in p.

As per 8.3.111, if P_2 starts with an s, it will not undergo retroflexion. So, for this case to apply, s has to be the 2^{nd} phoneme of the word P_2 and P_2 should start with a phoneme $q \in I_s$. Clearly, it requires that P_2 should start with a conjunct consonant from the set $\{ns, hs, ss, ss, ss\}$. The language does not allow such words, starting with this pattern. Internal sandhi rules explicitly eliminate the last four, and general sonority hierarchy rules out the first in syllable structure.

Case 3: So, we are left only with the case where the first phoneme p of P_2 is not in the set C_s and by external sandhi with P_1 , changes to the set C_s . Also, the second phoneme of P_2 should be s after external sandhi of P_1 and P_2 , so that the first phoneme $p \in C_s$ of P_2 will provide a condition for retroflexion in the second phoneme s. Let us now look into this case in further details, taking the two exhaustive cases, where the first phoneme of P_2 did not belong to C_s and external sandhi changed it into a phoneme in C_s , either consonant or vowel.

Let us consider the first possibility. Due to the language constraints, the consonant $p \in C_s$ has to be k and only k, because other consonants in set C_s do not occur before s or \mathfrak{s} , while starting a word. Let us consider the rules which can change a consonant $q \notin C_s$ to k.

8.2.30. coh kuh: A phoneme in the set $cu = \{c, ch, j, jh, \tilde{n}\}$ will be changed to the corresponding phoneme in the set $ku = \{k, kh, g, gh, \dot{n}\}$, if a phoneme in $praty\bar{a}h\bar{a}ra\ jhal$ follows. Though s belongs to the $praty\bar{a}h\bar{a}ra\ jhal$, this rule would not qualify for our analysis since this rule does not look at the phoneme preceding cu

and therefore, would have been applied on the original word itself and the original word would have the retroflexion.

The same reasoning can be applied to the following rule as well:

8.2.41. sadhoh kah si: A phoneme in the set $\{s, dh\}$ will be changed to k if s follows.

So, we will never have a case where external sandhi changes a consonant $q \notin C_s$ to a consonant $p \in C_s$, such that p is the first phoneme and s is the second phoneme of P_2 after external sandhi.

Let us now consider the second possibility, which is common enough and the language does not constrain the vowels to be used before s. However, the only possible cases here are if any vowel q in the set $A = \{a, \bar{a}\}$ is changed to a vowel $p \in C_s$ by external sandhi. Such a case arises. Consider the following example:

```
ka \ \dot{h} + asicat \rightarrow ka \ u + asicat \ (6.1.109 \ ato \ roraplut\bar{a}daplute)

ka \ \dot{u} + asicat \rightarrow ko + asicat \ (6.1.84 \ \bar{a}dguna\dot{h})

ko + asicat \rightarrow kosicat \ (6.1.105 \ enah \ pad\bar{a}nt\bar{a}dati).
```

At this stage, the o could have triggered the retroflexion in s. However, this application is prevented because of the following rule:

6.1.86 (satvatukorasiddhah): A single replacement of two sound elements is suspended when a replacement in s or insertion of tuk is to take place. Therefore, this case is prevented by the grammar itself.

We have now exhausted all the possibilities and from this discussion, we can safely say that the general case for $s \to s$ retroflexion is that the retroflexion does not happen across pada, even though the term $sam\bar{a}napade$ has not been used by $P\bar{a}nini$ for this case. There are certain rules, allowing retroflexion across the constituent words in a compound. These rules are treated as exceptions of the rule 8.3.111. Some of the examples are:

```
aṅguli – saṅga → aṅguliṣaṅga (8.3.80 samāse aṅguleḥ saṅgaḥ) agni – stoma → agniṣṭoma (8.3.82 agneḥ stutstomasomāḥ) mātṛ – svasṛ → mātṛṣvasṛ (8.3.84 mātṛpitṛbhyāṃ svasā).
```

These examples provide a finite list, which can easily be lexicalized. Retroflexion rules corresponding to the compounds are also discussed in 8.3.65 to 8.3.77. In tradition, these rules are also treated as exceptions of 8.3.111. In this section, there are specific rules for various verbs and preverbs, and each pair of preverb and verb is to be handled separately.

While we are discussing retroflexion, we will also like to draw the attention of the readers to a specific case of non-retroflexion in the case of *kurvanti*.

The Case of Kurvanti

A Sanskrit student who does not know Pāṇini's grammar by heart may find it difficult to give a quick answer as to why there is no retroflexion in kurvanti? As stated before, the standard rules for retroflexion $n \rightarrow p$ in Pāṇini are 8.4.1 and 8.4.2. The word kurvanti has an n preceded by r in a single word (8.4.1) and the intervening phonemes a and v fall in the set ap, matching the criteria of 8.4.2. Therefore, it appears as if the form kurvanti should undergo retroflexion. But it does not, the reason being two other rules stated in Pāṇini's grammar.

8.3.24 $na\dot{h}$ ca apadāntasya jhali: If n is followed by a consonant in $praty\bar{a}h\bar{a}ra$ jhal, it is replaced by \dot{m} .

8.4.58 anusvārasya yayi parasavarņah: If m is followed by a consonant in $praty\bar{a}h\bar{a}ra$ yay, it is replaced by the homogeneous nasal.

Consider the set T= consonants— $\{\{\dot{n},\ \tilde{n},\ \dot{n},\ n,\ m\}\cup\{y,\ v,\ r\}\}$, which denotes the phonemes in the $praty\bar{a}h\bar{a}ra$ set jhal. If n is followed by a phoneme from the set T, the rule 8.3.24 gets the application, bleeding the retroflexion rules. For the set $H=T-\{h,\ \dot{s},\ s,\ s\}$, the rule 8.3.24 feeds the rule 8.4.58. Specifically, in the case under consideration,

$$kurva(n)ti \xrightarrow{8.3.24} kurva(\dot{m})ti \xrightarrow{8.4.58} kurva(n)ti.$$

So, effectively no change took place. On the other hand, if it has to undergo retroflexion, the process would be:

$$kurvanti \xrightarrow{8.4.2} kurvanti$$

The second process does not correspond to the desired form kurvanti. There is no rule in Pāṇini's grammar that allows converting the form kurvanti to kurvanti.

What prevents one from following this wrong process is the control structure of $trip\bar{a}d\bar{\iota}$. The rules in $trip\bar{a}d\bar{\iota}$ are to be applied

sequentially. The rule 8.3.24 appears before 8.4.2 and, therefore, should be applied before 8.4.2. Once the form is $kurva\dot{m}ti$ by the application of 8.3.24, 8.4.2 does not get the application (bleeding by 8.3.24) and the process directly goes to 8.4.58 (feeding by 8.3.24), which converts $kurva\dot{m}ti$ back to kurvanti by changing \dot{m} to the homonasal of d, which is n in this case. ¹⁶

Now, let us have a look at how this process is implemented in the Heritage site. In addition to coding the rules 8.4.1 and 8.4.2, an additional criteria is employed for retroflexion. As per this criteria, the right-hand side of n is checked and only if it does not contain any consonant in $\operatorname{praty}\bar{a}h\bar{a}ra$ jhal , it can undergo retroflexion. Clearly, the retroflexion process is not exactly Pāṇinian and if asked to enumerate the rules leading to the form $\operatorname{kurvanti}$ in the Heritage system, it will definitely not list the rules 8.3.24 and 8.4.58, which actually cancel the effect of each other. However, it might be looked as a much more complete rule for the retroflexion condition.

5 Idiosyncracies of Phonology

5.1 Phonemic Variations

The Heritage engine computes on words and sentences represented as phonemic lists, with phonemes represented as integers, in the simplistic but efficient style of the Zen library. The user interface interprets user input in four common transliteration styles and immediately converts it to normalized words, where normalization, explained in footnote 2, replaces (non genuine) anusvāra by the equivalent nasal. All computed forms and morphemes are stored in this normalized form, avoiding useless non-determinism at segmenting time.

One important remark is that these stored forms are generally in terminal sandhi form, except that final r is preserved, and not turned into visarga. This is necessary to parse phrases such as punarapi, analysable as the sandhi of punar and api. Storing punah instead would prevent this analysis and allow the wrong *punopi.

We thank Malhar Kulkarni for his explanation of this intricate Pāninian process.

Gemination, allowing e.g. karmma for karma, is not allowed, although allowed (optionally) by $8.4.46.^{17}$ In that we follow the opinion of Śākalya. Degemination, for instance of $v\bar{a}rtt\bar{a}$ for $v\bar{a}rtt\bar{a}$, is allowed for a few lexical items. 19

The data structure of $praty\bar{a}h\bar{a}ra$ is not used in the Heritage machinery. The corresponding sets of phonemes are represented by unions of integer intervals.

5.2 Finer Phonemic Distinctions

The Heritage machinery makes finer phonemic distinctions than the tradition, in order to simplify some morpho-phonemic treatment. Specifically, we consider three versions of phoneme h and two versions of phoneme j.

Variants of Phoneme h

The standard (cerebral) version of phoneme h combines with t in sandhi to yield dh. Thus, for example, root $g\bar{a}h$ admits as past participle $g\bar{a}dha$. But sometimes gdh is obtained, like in dugdha for root duh, whose final h has kept a guttural character. And rarely

8.4.64 halah yamām yami lopaḥ: A phoneme in the pratyāhāra yam is deleted optionally if it is preceded by any consonant (hal) and is followed by another phoneme in the pratyāhāra yam. 8.4.65 jharaḥ jhari savarṇe: A phoneme in the pratyāhāra jhar is deleted optionally if it is preceded by any consonant and is followed by a homogeneous phoneme in the pratyāhāra jhar.

As an interesting observation, the two *pratyāhāra* sets, *yam* and *jhar* are exclusive and exhaustive to cover all the consonants. Thus, after any consonant, if two homogeneous consonants follow in the close proximity, the first of the two can optionally be deleted by these two rules.

 $^{^{17}}$ 8.4.46 acah rahābhyām dve: A phoneme in the pratyāhāra yar, when occurring in close proximity after a vowel followed by r or h, is optionally doubled.

 $^{^{18}}$ 8.4.51 sarvatra śākalyasya: In the opinion of Śākalya, there is no doubling.

¹⁹ In $trip\bar{a}d\bar{\imath}$, there are two optional rules regarding degemination:

ddh is obtained, like in naddha for root nah, whose final h has kept a dental character.

These sandhi rules are handled in the Heritage site by a finer phoneme distinction than in the traditional view, with extra phonemes h' and h", subject to the following sandhi rules:

 $h' + t \rightarrow gdh$ and $h'' + t \rightarrow ddh$ (and other similar rules for consonants other than 't').

As per $P\bar{a}$ nini's grammar, the general rule for sandhi between h and any phoneme in jhal is

8.2.31 hah dhah: h is replaced by dh if any phoneme in jhal follows. For the specific case of h followed by t, this rule feeds some other rules, yielding the following derivation sequence:

$$\begin{array}{c} h + t \xrightarrow{8.2.31} dh + t \xrightarrow{8.2.40} ^{20} dh + dh \xrightarrow{8.4.41} dh + \\ dh \xrightarrow{8.3.13} ^{21} dh. \end{array}$$

However, there are rules, exception to 8.2.31, which in addition to looking at the phonemes, also look for the lexical criteria such as the actual root word and the sandhi rule for the h appearing in such roots is different. For example:

8.2.32 $d\bar{a}deh$ $dh\bar{a}toh$ ghah: For the roots starting with d, h is replaced by gh if any phoneme in jhal follows. Roots such as $\{dah, dih, duh\}$ fall in this category.

8.2.33 $v\bar{a}$ druhamuhaṣṇuhaṣṇihām: For the roots $\{druh, muh, sṇih, ṣṇuh\}$, h is optionally replaced by gh if any phoneme in jhal follows.

For the specific case of h followed by t in these roots, these rules can yield the following derivation sequence:

²⁰ 8.2.40 *jhaṣaḥ tathoḥ dhaḥ adhaḥ*: The phonemes t and th are replaced by dh, if they follow after a phoneme in the set *jhaṣ*, with the exception of the root $dh\bar{a}$.

 $^{^{21}}$ 8.3.13 dhaḥ dhe lopaḥ: The phoneme dh is deleted, when followed by dh.

It is to be noted that the rule 8.3.13 has been applied here after 8.4.41 and, therefore, is an exception of the asiddha principle. These cases of $\bar{a}\acute{s}ray\bar{a}t$ siddham have been discussed in detail by Cardona in (2011).

 $h+t \xrightarrow{8.2.32|8.2.33} gh+t \xrightarrow{8.2.40} gh+dh \xrightarrow{8.4.53}^{22} g+dh$. 8.2.34 nahaḥ dhaḥ: 'h' of the root nah is replaced by 'dh' if any phoneme in jhal follows. For the specific case of h followed by t in nah, this rule can yield the following derivation sequence:

$$h + t \xrightarrow{8.2.34} dh + t \xrightarrow{8.2.40} dh + dh \xrightarrow{8.4.53} d + dh.$$

In the Heritage system, the phoneme h corresponding to the rules 8.2.32 and 8.2.33 is denoted by h', while that corresponding to 8.2.34 is denoted by h''.

Variants of phoneme j

Similarly, phoneme j in its usual guttural version combines with t in sandhi to yield kt. Thus yukta for past participle of root yuj. But sometimes it acts similarly to the sibilant \acute{s} and yields rather st. Thus mrsta for past participle of root mrj. The Heritage site recognizes this distinction with a variant j' such that j' + $t \rightarrow st$ in order to accommodate forms of roots such as mrj. The roots concerned are $bhr\bar{a}j$, mrj, yaj, $r\bar{a}j$, vraj, and srj.

As per Pāṇini's grammar, the general rule for sandhi between j and any phoneme in jhal is:

8.2.30 coh kuh: A phoneme belonging to the set cu (palatals) is replaced by the corresponding phoneme in the set ku if any phoneme in jhal follows or at the end of a word.

Under specific conditions, this rule can feed another rule in the grammar:

8.4.55 khari ca: A phoneme belonging to the set jhal is replaced by the corresponding phoneme in set car, if followed by a phoneme in set khar.

$$khar = \{kh, ph, ch, th, th, c, t, t, k, p, ś, s, s\}$$

 $car = \{c, t, t, k, p, ś, s, s\}.$

²² 8.4.53 *jhalāṃ jaś jhaśi*: A phoneme in the set *jhal* is replaced by a phoneme in the set *jaś*, if followed by a phoneme in the set *jhaś*. In effect, this rule replaces a phoneme in the set *jhal* by the third letters, if followed by fourth letters of a group.

In effect, the rule 8.4.55 changes the second, third and fourth phonemes of the sound set to its first phoneme, if followed by a phoneme in set *khar*. So, in general, when j combines with t, both the rules 8.2.30 and 8.4.55 apply, yielding the following:

$$j + t \xrightarrow{8.2.30} g + t \xrightarrow{8.4.55} k + t.$$

However, there is an exception to rule 8.2.30:

8.2.36 vraścabhrasjasrjamrjayajarājabhrājachaśām ṣaḥ: The last phoneme of the roots vraśc, bhrasj, srj, mrj, yaj, rāj, bhrāj and roots ending in ch or \acute{s} are substituted by \acute{s} if any phoneme in jhal follows or at the end of a word.

Focusing on the combination of j and t, this rule gives an exception for the roots $\{bhrasj, srj, mrj, yaj, r\bar{a}j, bhr\bar{a}j\}$ and changes the j to s instead of g, as specified by the general rule 8.2.30. The phoneme s before t then feeds the following rule:

8.4.41 $stun\bar{a}$ stuh: A phoneme in the set stu ($\{s, t, th, d, dh, n\}$) is substituted by the corresponding phoneme in the set stu ($\{s, t, th, d, dh, n\}$), if it occurs before or after a phoneme in the set stu.

Together, these rules yield the following *sandhi* rule for these specific roots:

$$j + t \xrightarrow{8.2.36} s + t \xrightarrow{8.4.41} s + t$$

In the Heritage system, the phoneme j corresponding to the rule 8.2.36 is denoted by j'.

The variant phonemes which are dealt with in the Heritage engine may be justified by comparative linguistics, as remnants from earlier language substratum. They yield a more unified *sandhi* processing. The Pāṇinian tradition recognizes them indirectly, by giving specific rules for specific roots, but the end result is equivalent.

6 Soundness and Completeness

Evaluation of a computational linguistics platform is usually discussed in terms of the parameters of precision and recall, measuring the quality of response to a query. For a query admitting rv relevant

answers, let rt be the number of retrieved answers returned by the platform and right be the number of retrieved relevant answers. The precision is defined as right/rt, the recall as right/rv. The recall measures the completeness of the system. A low recall means that the system performs badly, in not recognizing correct sentences. Increasing the recall usually decreases the precision; we say that the system overgenerates, its correct answers being drowned in irrelevant ones.

We shall not give here precise figures of precision and recall for the Heritage platform, for lack of a statistically significant tagged corpus consistent with our system of tags and our assignment of homonymy indexes. Furthermore, many evaluation criteria may be envisioned, for the generational aspect as well as for the recognition aspect.

We shall however discuss the known incompletenesses of the system with respect to the $P\bar{a}ninian$ framework, and try to assess their statistical relevance. Before discussing recall, i.e. completeness, we discuss the mechanisms which limit overgeneration.

6.1 Curbing Overgeneration

Direct inversion of sandhi on a non-trivial sentence would result in literally millions of potential segmentations, most of them being sterile, with segments un-recognizable as padas. We curb this overgeneration by having our segmenter lexicon-directed non-compound padas are stored in pre-generated databases of forms arranged in lexical categories, compounds are analysed recursively as sequences of lexicalized forms, and the lexical analyser search is constrained by the phase transitions, which reflect the morphological geometry. Still, certain situations lead to important branching factors of the non-deterministic search. For instance, the form $par\bar{a}$ leads to a potential 164 legal choices, as explained in Huet (2005).

In order to curb overgeneration, a semantic filter is applied to the potential segmentations, using a simplified semantic roles $(k\bar{a}raka)$ analysis. This mechanism has been described in Huet (2007). Every segmentation candidate is expanded into the set of all its tagged interpretations. Each tagged interpretation is given a penalty, reflecting the lack of match between the semantic roles of the substantive items and the regime of the verbal items (sakar-maka/akarmaka), determining its expectations $(\bar{a}k\bar{a}nks\bar{a})$. In this

simplified analysis, only the roles of agent (kartr) and goal/patient (karma) are taken into account. Every segmentation is then assigned as penalty the minimum penalty of its interpretations, and the system truncates all solutions below a given threshold. This semantic filtering reduces drastically the number of returned solutions, with a clear increase in precision, without too much loss in recall.

Two remarks are in order. The first one is that the inner structure of compounds is not relevant to this simplified semantic analysis, since no inner component of a compound may feed direct roles in Sanskrit. Note that the examples of asamartha compounds exhibited by Gillon (1993, 2002) concern only cases where the inner component of a compound has an exocentric argument as Bhartrhari puts it, this component expresses a relation. This vindicates our flat analysis of pre-compounds, since a pre-compound with n+1 components may correspond to C_n possible binary compounds, where the Catalan number C_n is exponential in n, as noted earlier.

The second remark is that, none the less, certain good segmentations may be discarded because of ellipses (typically, the agent may be implicit from the context as the current topic). This means that the $k\bar{a}raka$ analysis ought to be done at the level of discourse, rather than individual sentences.

Another device that curbs overgeneration in the Heritage platform is the regular nature of the segmenting automaton, the phases of the lexical analyser being constrained by a regular expression limiting the recursion. Thus small lexical items such as root substantives (-pa, -ga, $-n\bar{\imath}$, etc.) can generally occur only in Ifc position (right component of a compound), limiting drastically their combinatorics.

Still another important device is that vocatives are allowed only as terminal strings of the various segments. This avoids frequent ambiguities of stems in -a as bare stems usable as left components of compounds and as vocative interjections. While $P\bar{a}nini$ does not exclude the vocatives from undergoing sandhi, it is a rare phenomena to obtain the vocatives as a non-final segment and it can be justified by the prosody constraints. It still adds to the incomplete-

nesses of the system, as the systems cannot recognize the vocatives which appear as a non-final segment.²³

Finally, two distinct modes of the Sanskrit Reader are offered to the user. The so-called Simplified mode uses the simple state diagram shown in Fig. 1. It forbids vocatives and does not use the productive devices of participle generation and privative compounds such stems must be lexicalized in order to be recognized. The simplified mode is adequate for simple sentences and has very good precision (often, a unique segmentation is proposed). For more complex sentences, the Complete mode, using the full databank of participles and the complete analysis of privative compounds, is necessary. Let us now indicate in which ways this "Complete" mode is still incomplete.

6.2 Known Incompletenesses

One basic limitation of the Sanskrit Heritage engine is that it relies on lexicalization of atomic items. At the time of writing, its lexicon lists 588 roots and 9,200 non-compound stems. 1,560 combinations of preverbs and roots provide as many verbs, each allowed in all finite conjugation paradigms available in Classical Sanskrit (present, imperfect, optative, imperative, future, perfect, aorist/injunctive), plus infinitive and absolutive (with transitions allowing absolutives in $-tv\bar{a}$ only from roots, and those in -ya only when a preverb is present). Ten forms of participles are generated. Secondary conjugation stems (causative, desiderative and intensive) are lexicalized on demand. Other primary derivatives (krdantas) are available only when lexicalized. The same is true of secondary derivatives, which is certainly a strong limitation, that will have to be lifted at some future point, since many taddhita suffixes are very productive ($-t\bar{a}$, -tva, -tr, etc.)

The segments containing the non-final vocatives are shown in bold. The first verse also gives an example $p\bar{a}rth\bar{a}sti$, where the vocatives can undergo sandhi.

²³ In the $Bhagavadg\bar{\imath}t\bar{a}$ (Bh.G.), for instance, there are at least three verses where the vocatives appear as a non-final segment:

⁻ Bh.G. 3.22: na me pārthāsti kartavyam triṣu lokeṣu kiñcana

⁻ Bh.G. 3.28: tattvavit tu **mahābāho** gunakarmavibhāgayoh

⁻ Bh.G. 5.3: nirdvandvo hi **mahābāho** sukham bandhāt pramucyate.

A few lexical items have been omitted from the lexicon because they caused too much overgeneration. For instance, the substantive vi (bird), for obvious clash with the homophonic particle or the rare substantive atra (food) for similar confusion with the adverb. Also the substantive $\bar{a}ya$ for its conflict with dative forms as well as the form $n\bar{a}$ (nominative of nr). It is hoped that such difficulties will be overcome at some stage, when the system will be tuned by statistical training.

The analysis given in Section 3 ought to convince the reader that most productive schemes are correctly dealt with. Non-productive schemes (*aluk*, retroflexion) assume lexicalization of the corresponding compound stem.

The strongest limitation concerns substantival verbs, such as *yantrayati*. At present, such verbal stems must be lexicalized. Dealing with substantival verbs in all generality, including for past forms with augment, seems out of reach with the present technology.

Sanskrit is a complex language, whose literature covers many styles over three millenia. It has an extremely productive morphology, allowing the generation of extravagant items such as ci- $candr\bar{\imath}y\bar{\imath}sak\bar{a}yam\bar{a}nena$ (Dīkṣita et. al., 1905). Current technology does not permit the automatic analysis of such complex forms. However, compounds with any number of components, such as $pravaranrpamukutamanimar\bar{\imath}cimanjar\bar{\imath}cayacarcitacaranayugalah$ (attested in Pañcatantra), do not pose particular problems.

7 Conclusion

The Sanskrit Heritage reader recognizes simple sentences from Classical Sanskrit. In Simplified mode (Fig. 1), it is precise but incomplete. In Complete mode (Fig.2), it exhibits a good recall of sentences whose vocabulary is covered by the lexicon. The system is fast and robust. This paper made an attempt at relating its operations to those of the $P\bar{a}ninian$ tradition. Although the details of derivation vary, there seems to be no major discrepancy. The most difficult problems left to solve are the analysis of denominative verbs, and the proper treatment of ellipsed agents and double accusative verbs (dvikarmaka).

References

- 1. von Böhtlingk, O. *Pāṇini's Grammatik*. Language and Linguistics Series. Motilal Banarsidass, Delhi, 1998.
- 2. Cardona, G. *Pāṇini: His Work and Its Traditions*. Motilal Barnasidass, Delhi, 1988.
- 3. Cardona, G. *Pūrvatrāsiddham and Āśrayāt Siddham*. D.K. Printworld, Delhi, 2011.
- 4. Dīkṣita, B., S. Vasu, and B. Das Basu. The Siddhānta Kaumudi of Bhattoji Dikshita. Number v. 3 in The Siddhanta Kaumudi of Bhattoji Dikshita. The Panini Office, 1905.
- 5. Gillon, B.S. 'Bartrhari's solution to the problem of asamartha compounds. Études Asiatiques/Asiatiche Studien, 47,(1): 117–133, 1993.
- 6. Gillon, B.S. *Indian linguistic studies: Festschrift in Honour of George Cardona*; eds. Madhav M. Deshpande et Peter E. Hook, chapter 'Bartṛhari's Rule for Unexpressed Kārakas: The Problem of Control in Classical Sanskrit. Motilal Banarsidass, Delhi, 2002.
- 7. Gillon, B.S. 'Exocentric (Bahuvrīhi) Compounds in Classical Sanskrit. In G. Huet and A. Kulkarni, eds., *Proceedings, First International Symposium on Sanskrit Computational Linguistics*, pp. 1–12, 2007.
- 8. Gillon, B.S. 'Tagging Classical Sanskrit Compounds. In A. Kulkarni and G. Huet, eds., *Sanskrit Computational Linguistics 3*, pp 98–105. Springer-Verlag LNAI 5406, 2009.
- 9. Huet, G. *The Zen Computational Linguistics Toolkit*. Technical report, ESSLLI Course Notes, 2002.
- 10. Huet, G. 'Towards Computational Processing of Sanskrit. In *International Conference on Natural Language Processing (ICON)*, 2003.
- 11. Huet, G. 'A Functional Toolkit for Morphological and Phonological Processing, Application to a Sanskrit Tagger. *J. Functional Programming*, 15,(4): 573–614, 2005.
- 12. Huet, G. Themes and Tasks in Old and Middle Indo-Aryan Linguistics, eds. Bertil Tikkanen and Heinrich Hettrich, chapter 'Lexicondirected Segmentation and Tagging of Sanskrit, pp. 307–25. Motilal Banarsidass, Delhi, 2006.
- 13. Huet, G. and B. Razet. 'The Reactive Engine for Modular Transducers. In K. Futatsugi, J.-P. Jouannaud, and . Meseguer, eds., Algebra, Meaning and Computation, Essays Dedicated to Joseph A. Goguen on the Occasion of His 65th Birthday, pp. 355–74. Springer-Verlag LNCS, vol. 4060, 2006.
- 14. Huet, G. 'Shallow Syntax Analysis in Sanskrit Guided by Semantic Nets Constraints. In *Proceedings of the 2006 International Workshop on Research Issues in Digital Libraries*, New York, NY, USA, 2007. ACM.

- 15. Huet, G. 'Sanskrit Segmentation. XXVIIIth South Asian Languages Analysis Roundatable, University of Denton, Texas, 2009.
- 16. Joshi, S. and J. Roodbergen. Patañjali's Vyākaraṇa-Mahābhāṣya Sthānivadbhāvāhnika: Introduction, Text, Translation and Notes. Number v. 1 in Research Unit Series. Bhandarkar Oriental Research Institute, 1990.
- 17. Joshi, S. and J. Roodbergen. *The Aṣṭādhyāyī of Pāṇini with Translation and Explanatory Notes*. Number v. 11 in The Aṣṭādhyāyī of Pānini. Sāhitya Akādemī, 2004.
- 18. Kaplan, R.M. and M. Kay. 'Regular Models of Phonological Rule Systems. *Computational Linguistics*, 20,(3): 331–78, 1994.
- 19. Kielhorn, F. *The Paribhashendusekhara of Nagojibhatta*. Parimal Publications, Delhi (reprint), 1871.
- 20. Kiparsky, P. 'On the architecture of Pāṇini's grammar. In G. Huet, A. Kulkarni, and P. Scharf, eds., Sanskrit Computational Linguistics 1 & 2. Springer-Verlag LNAI 5402, 2009.
- 21. Kulkarni, A. and D. Shukl. 'Sanskrit Morphological Analyser: Some Issues. *Indian Linquistics*, 70(1-4): 169–77, 2009.
- 22. Monier-Williams, M., E. Leumann, and C. Cappeller. A Sanskrit-English Dictionary: Etymological and Philologically Arranged with Special Reference to Cognate Indo-European Languages. Asian Educational Services, 1999.
- 23. Munśī, O., E. Unithiri, and N. Unithiri. *Dhāturūpaprapañca (A Dictionary of All the Forms of All the Roots in Sanskrit)*. Number pt. 2 in Calicut University Sanskrit Series. Publication Division, University of Calicut, 2004.
- 24. Scharf, P. 'Levels in $P\bar{a}nini$'s $A\underline{s}t\bar{a}dy\bar{a}y\bar{\imath}$. In A. Kulkarni and G. Huet, eds., Proceedings, Third~International~Symposium~on~Sanskrit~Computational~Linguistics, volume LNAI 5406, pp. 66–77. Springer, 2009.
- Sharma, A., K. Deshpande, and D. Padhye. Kāśikā: A Commentary on Pāṇini's Grammar. Sanskrit Academy series. Sanskrit Academy, Osmania University, 2008.
- 26. Sharma, R. The Aṣṭādhyāyī of Pāṇini: English translation of adhyāyas seven and eight with Sanskrit text, transliteration, word-boundary, anuvṛtti, vṛtti, explanatory notes, derivational history of examples, and indices. The Aṣṭādhyāyī of Pāṇini. Munshiram Manoharlal Publishers, Delhi, 2003.
- 27. Vasu, S. *The Aṣṭādhyāyī of Pāṇini*. Motilal Banarsidass, Delhi, 1980.

Appendix A

Let us consider the derivation of future passive participles (pfp) ending in ya and we will draw parallel between the Heritage system and the rules in $Ast\bar{a}dhy\bar{a}y\bar{\imath}$.

$A \underline{s} t \bar{a} dh y \bar{a} y \bar{\imath}$ Treatment

In $A \not s \not t \bar a dhy \bar a y \bar i$, there are three different $k \not r t y a$ suffixes, which are used for the pfp ending in ya: $\{yat, kyap, nyat\}$. The treatment of this pfp formation can be considered a two-stage process of first identifying the proper suffix out of the three, and second stem changes according to the suffix.

Identifying the Proper Suffix

Main rules in Pāṇini's grammar corresponding to identifying the pfp suffixes are:

- 3.1.97 acah yat: The suffix yat is added to the roots ending in a vowel.
- 3.1.98 poh adupadhāt: The suffix yat is added to the roots ending in a vowel and having a as the penultimate phoneme.
- 3.1.109 $etistuś\bar{a}sv_r drjuṣah kyap$: The suffix kyap is applied to the roots $\{i, stu, ś\bar{a}s, vr, dr, jus\}$.
- 3.1.110 $rdupadh\bar{a}t$ ca ak!picrteh: The suffix kyap is applied to the roots having short r in the penultimate, with the exception of klp and crt.
- 3.1.124 *rhalor nyat*: The suffix *nyat* is applied to the roots ending in r or a consonant.

Together, these rules along with some specific rules in 3.1.97 to 3.1.132 determine which of the three suffixes should be applied to a root.

Stem Changes

Once the suffix is decided, the rules in the grammar corresponding to the stem changes are applied. For example, for suffix yat:

6.1.45 $\bar{a}t$ ecah upadeśe aśiti: For a root ending with one of the $\{e, ai, o, au\}$ in the upadeśa, the last phoneme is substitued by \bar{a} , if a suffix not having an indicatory ś follows. 6.4.65 $\bar{\imath}t$ yati: If suffix yat follows an aṅga ending in \bar{a} , the last \bar{a} is replaced by $\bar{\imath}$.

Because yat is an $\bar{a}rdhadh\bar{a}tuka$ suffix, the anga gets the guna by 7.3.84 $(s\bar{a}rvadh\bar{a}tuk\bar{a}rdhadh\bar{a}tukayoh)$.

For suffix kyap, the main stem changes occur because of the following rule:

6.1.71 hrasvasya piti k_r ti tuk: When a short vowel is followed by a k_r t suffix having an indicatory p, it gets an insertion of tuk (t).

Because kyap has an indicatory k, it does not cause guna because of 1.1.5 kkniti ca.

For suffix nyat, the main stem changes occur because of the following rules:

7.2.115 $acah \tilde{n}niti$: An anga ending in a vowel gets vrddhi before a suffix having an indicatory \tilde{n} or an indicatory \tilde{n} .
7.2.116 $atah upadh\bar{a}y\bar{a}h$: The penultimate a of an anga gets vrddhi before a suffix having an indicatory \tilde{n} or an indicatory \tilde{n} .

Because nyat is an $\bar{a}rdhadh\bar{a}tuka$ suffix, when the anga does not satisfy the above two rules, it gets the guna by 7.3.86 ($pugantalagh\bar{u}padhasya$ ca).

After the stem gets $v_r ddhi$, there are some phonetic changes because of the following rule:

7.3.52 cajoh ku ghinyatoh: The final c or j of an anga are replaced by k and g, respectively, if a suffix having an indicatory gh or the suffix nyat follows.

There are some exceptions to 7.3.52, such as:

7.3.59 na $kv\bar{a}de\dot{p}$: The change from c or j to k and g does not take place if the root starts with a velar.

7.3.66 $yajay\bar{a}carucapravacarcah$ ca: The change from c or j to k and g does not take place for the roots $\{yaj, y\bar{a}c, ruc, pravac, rc, tyaj\}$. The inclusion of the root tyaj is because of a $v\bar{a}rtik\bar{a}$ on this rule.

Heritage Treatment

The Heritage system does not implement pfp as a two-stage process as described above but the rules for identifying suffix and stem changes are implemented together. Given a root, the function **pfp-ya** builds the stem pfp-stem. Once the pfp-stem is built, the condition of phonemic change due to 7.3.52 is checked by a function **fix-pfp-ya** and the specific exceptions of 7.3.52 are encoded in the function **palatal-exceptions**. We will give a brief pseudocode of these three functions along with the reference to Pāṇini's rules on pages 170 and 171. The function pfp-ya takes care of all the roots, which take yat and nyat suffixes and the roots ending in a consonant, which take kyap suffix. The roots which take kyap suffix and end in a vowel, are dealt with specifically in a separate function, which also generates the supplementary pfp forms.

It is clear from the pseudocode that every operation in the Heritage system emulates the corresponding operations in $P\bar{a}nini$'s grammar. A single line of code (phonetic rewrite rule) corresponds to one or many rules ($s\bar{u}tra$) in $Ast\bar{a}dhy\bar{a}y\bar{\imath}$; as discussed in section 1, these rewrite rules could be used as a guide to the generation of the $P\bar{a}nini$ derivation.

```
\begin{array}{c} \textbf{pfp-ya (root)} \\ \textbf{pfp-stem} = \\ & ( ** if \ root \ ends \ in \ a \ vowel \ **) \\ \textbf{match last phoneme of root with} \\ & a \rightarrow \textbf{root} \ (3.1.97) \\ & \bar{a} \mid ai \mid o \mid au \rightarrow \textbf{replace last with } e \ (3.1.97, \ 6.1.45, \ 6.4.65, \ 7.3.84) \\ & i \mid \bar{\imath} \mid u \mid \bar{u} \rightarrow guna \ \text{of root} \ (3.1.97, \ 7.3.84) \\ & \mathbf{r} \rightarrow vrddhi \ \text{of root} \ (3.1.124, \ 7.2.115) \\ & ( ** if \ root \ ends \ in \ a \ consonant \ **) \\ \textbf{match last and penultimate phoneme with} \\ & \{\mathbf{x},a\} \rightarrow \textbf{if} \ \mathbf{x} \in \{p,ph,b,bh,m\} \ \text{then root} \ (3.1.98) \\ & \quad \textbf{else replace} \ a \ \text{by} \ \bar{a} \ (3.1.124, \ 7.2.116) \\ & \{\mathbf{x},\mathbf{r}_{\circ}\} \rightarrow \textbf{root} \ (3.1.110) \\ & \quad \textbf{else} \rightarrow guna \ \text{of root} \ (3.1.124, \ 7.3.86) \\ \end{array}
```

fix-pfp-ya (root, pfp-stem) pfp-stem-new = if palatal-exception (root) \rightarrow pfp-stem else if pfp-stem starts with a velar \rightarrow pfp-stem (7.3.59) else match the last phoneme of pfp-stem with $c \rightarrow$ replace c with k (7.3.52) $j \rightarrow$ replace j with j (7.3.52) else \rightarrow pfp-stem

```
palatal-exception (root)
match root with
aj \mid vraj \rightarrow \text{True } (7.3.60)
yaj \mid y\bar{a}c \mid ruc \mid rc \mid tyaj \rightarrow \text{True } (7.3.66)
srj \mid vrj \mid prc \rightarrow \text{True } (** because these roots take kyap (3.1.110)
**)
else \rightarrow \text{False}
```