CS21004 - Tutorial 5

February 2, 2019

Instructions: For the problems with (To submit), please write the answers neatly in loose sheets and submit to the TA before the end of the tutorial.

1. Show the following languages are not regular language using Pumping Lemma

   (a) L = $\{a^n|\ n$ is a perfect square $\}$
   Solution: Let the language be regular with a pumping length m. First of all note that it has to be the case that $m > 1$. We can easily pick any string of length 1 $(= m)$ that can be pumped to generate a string of length 2,3,... which are not in L. So we can safely assume that pumping length for the language is $m > 1$. Now we pick a string $w = a^{m^2}$ of length$> m$. For any decomposition $w = xyz$ of w, the middle part $y = a^k$ satisfies $1 \leq k \leq m$ by pumping lemma (clause 1 and 2). Pumping down we have a^{m^2-k}. Note that
   $k \leq m$
   $\Rightarrow k + 1 \leq m + 1 < 2m$
   $\Rightarrow k < 2m - 1$
   $m^2 - k > m^2 - 2m + 1$
   $m^2 - k > (m - 1)^2$
   Hence, the resulting string is not having a number of a-s which is a perfect square.

   (b) L=$\{0^i x \mid i \geq 0,\ x \in \{0,1\}^*$ and $|x| \leq i\ \}$ (**To submit**)
   Solution: Let pumping length be p. Note $\sigma = 0^p1^p \in L$ setting $i = p$ and $x = 1^p$. Hence, by pumping lemma, $\exists u, v, w$ such that

      i. $|v| > 0$
      ii. $0^p1^p = uvw$ with $u = 0^m$, $v = 0^n$ where $m + n \leq p$
      iii. $uv^kw \in L \forall k \geq 0$

   Setting $k = 0$, $uw \in L \Rightarrow 0^{p-n}1^p \in L$ which is not true. This is because $0^{p-n}1^p$ cannot be in L.

   (c) L =$\{w|\ n_a(w) \neq n_b(w)\}$ (**To submit**)
   Solution: Let L be regular. Then L' = $\{w|\ n_a(w) = n_b(w)\ \}$ is also regular (complementation). With pumping length p consider 0^p1^p.

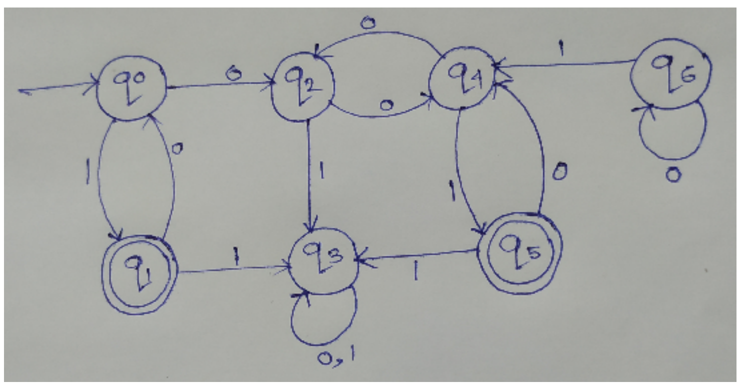
(d) L $= \{0^n(12)^m : n \geq m \geq 0\}$ (**To submit**)

Solution: Let L be regular language and p be the pumping length for L given by the pumping lemma. Let $s = 0^p(12)^p$. Then s can be split into xyz satisfying the conditions of the pumping lemma. By the condition of pumping lemma $|xy| \leq p$ y must contain only 0s. Pumping lemma states $xy^iz \in L$ even when $i = 0$. So, let us consider the string $xy^0z = xz$. Removig y decreases the number of 0s in s. Therefore, xz cannot have at least as many 0s as (12)s. Hence $xy \notin L$. This contradicts our initial assumption.

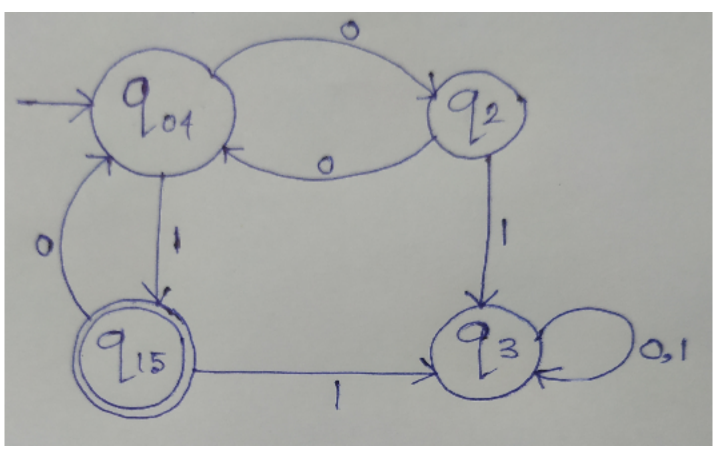
(e) L $= \{w :\ w$ has balanced parentheses$\}$(Home)

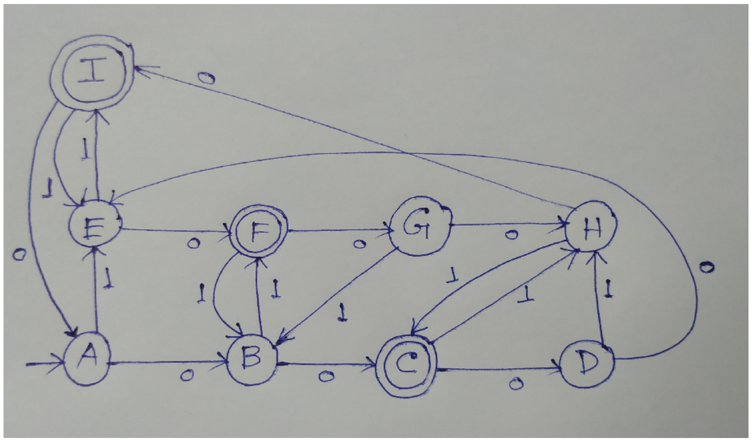
(f) L $= \{a^{n!} | n \geq 0\}$ (Home)

2. Minimize the following DFAs (**Submit the second one**)

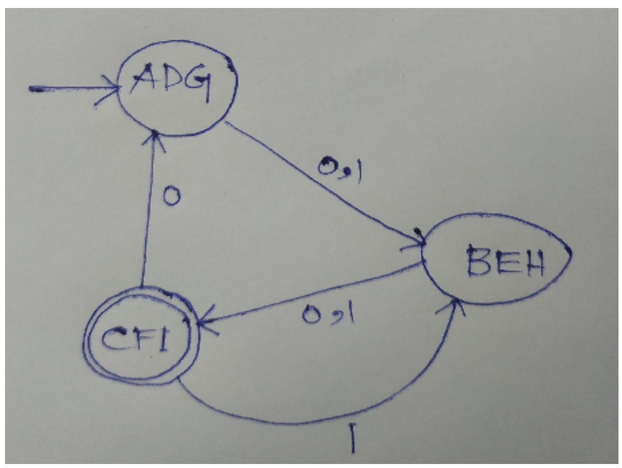


Solution:





Solution:



3. Provide an algorithm for converting a left linear grammar to a right linear grammar. (Home)

4. Show that the family of regular languages is closed under symmetric difference. (Home)