

**CS21201 Discrete Structures**  
**Tutorial**  
**Generating Functions**

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1. Let  $F_n, n \geq 0$ , denote the Fibonacci numbers. Prove that  $\sum_{n \in \mathbb{N}_0} \frac{F_n}{2^n} = 2$ .
2. Let  $l_n$  be the number of lines printed by the call  $f(n)$  for some integer  $n \geq 0$ .

```
void f ( int n )
{
    int i, j;
    printf("Hi\n");
    if (n == 0) return;
    for (i=0; i<=n-1; ++i)
        for (j=0; j<=i; ++j)
            f(j);
}
```

- (a) Let  $L(x) = l_0 + l_1x + l_2x^2 + \dots + l_nx^n + \dots$  be the generating function of the sequence  $l_0, l_1, l_2, \dots$ . Prove that  $L(x) = \frac{1-x}{1-3x+x^2}$ .
  - (b) Derive an explicit formula for  $l_n$  (valid for all  $n \geq 0$ ) from the generating function  $L(x)$ .
3. Let  $a_0, a_1, a_2, \dots, a_n, \dots$  be the sequence generated by  $\sum_{r \in \mathbb{N}} \frac{x^r}{1-x^r}$ . Denote by  $p_n$  the parity of  $a_n$ , that is,  $p_n = 0$  if  $a_n$  is even and  $p_n = 1$  if  $a_n$  is odd. Determine all  $n \in \mathbb{N}$ , for which  $p_n = 1$ . Justify.
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