

CS21201: Discrete Structures

Autumn 2024

Tutorial: Set Sizes

1. (a) A real number r is said to be algebraic if r is a root of a polynomial with rational coefficients. If r is not ~~rational~~ algebraic, r is said to be transcendental. Show that the set of all transcendental numbers is uncountable.
(b) Let T denote the set of infinite binary sequences. We say that two binary sequences A_1 and A_2 are equivalent if they have the same tail end. For instance $1001111\dots$ and $111111\dots$ are equivalent.
Prove or disprove: For any binary sequence A , there are only countably many binary sequences equivalent to A .
2. Prove that the union of two sets equinumerous with \mathbb{R} is also equinumerous with \mathbb{R} .
3. Let A be an infinite set.
 - (a) Prove that there is a map $A \rightarrow A$ which is injective but not surjective.
 - (b) Prove that there is a map $A \rightarrow A$ which is surjective but not injective.