Discrete Structures 2024

Sets, Relation, Function - Tutorial Problems

September 12th 2024

1. $S = \{(1,2),(2,1)\}$ is a binary relation on the set $A = \{1,2,3\}$. Is it irreflexive? Add the minimum number of ordered pairs to S to make it an equivalence relation. What is the modified S?

2. Let $m,n \in \mathbb{Z}^+$ with $1 < n \leq m$. Then, $S(m+1,n) = S(m,n-1) + n \cdot S(m,n)$, where $S(m,n)$ denotes a Stirling number of the second kind. Also, prove $O(m,n) = n! \cdot S(m,n)$, where O(m,n) is the number of onto functions in $f : A \to B$.

3. Give an example of a poset A and a non-empty subset S of A such that S has lower bounds in A, but the greatest lower bound(S) does not exist.

4. How many permutations of the 10 digits either begin with the 3 digits 987, contain the digits 45 in the fifth and sixth positions, or end with the 3 digits 123?

5. Define a relation ρ on $A = \mathbb{Z} \times \mathbb{N}$ as $(a,b)\rho(c,d)$ if and only if $ad = bc$. Prove that ρ is an equivalence relation. Argue that A/ρ is essentially the set $\mathbb{Q}$ of rational numbers. In abstract algebra, we say that $\mathbb{Q}$ is the field of fractions of the integral domain $\mathbb{Z}$. The equivalence class $[(a,b)]$ is conventionally denoted by a/b.