Discrete Structures 2024

Pigeonhole Principle - Tutorial Problems

September 5, 2024

1. Prove that in any group of 10 distinct positive integers between 1 and 50, there are at least two numbers whose difference is at most 5.

2. A repunit is an integer of the form $111\ldots1$. Prove that any $n \in \mathbb{N}$ with $\gcd(n, 10) = 1$ divides a repunit.

3. Show that there exists an integer n such that $0 < \sin n < 2^{-2022}$.

4. Let ξ be an irrational number. Prove that given any real $\varepsilon > 0$ (no matter how small), there exist infinitely many pairs of integers a, b such that $0 < a\xi - b < \varepsilon$.

5. Let $n \geq 2$ be an integer. You choose n distinct integers from the set $\{1, 2, 3, \ldots, n^2 - 1\}$. Prove that there must be two of the chosen integers (call them x and y) satisfying $0 < \sqrt{x} - \sqrt{y} < 1$.