

**CS21201 Discrete Structures**  
**Tutorial**  
**Recursive Constructions, Loop Invariance**

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1. What does the following function return on input  $n$ ? Also argue that the function terminates for  $n > 1$ .

```
int h ( int n ) {
    if ( n <= 0 ) return -1; /* Error condition */
    if ( n % 2 == 1 ) return 0; /* n is odd */
    return 1 + h(n*(n+1)/2); /* n is even */
}
```

2. What does the following function return upon the input of two positive integers  $a, b$ ? Prove it.

```
int f(int a, int b) {
    int x, y, u, v;
    x = u = a;
    y = v = b;
    while (x != y) {
        if (x > y) {
            x = x - y;
            u = u + v;
        } else {
            y = y - x;
            v = u + v;
        }
    }
    return (u + v) / 2;
}
```

3. Let  $S$  be the subset of the set of ordered pairs of integers defined recursively by  
Basis step:  $(1, 2) \in S$ .

Recursive step: If  $(a, b) \in S$ , then  $(a + b, b) \in S$ ,  $(a - b, b) \in S$  and  $(b, a) \in S$ .

- a. Does  $S$  contain  $(20, 23)$ ?
  - b. Does  $S$  contain  $(357, 819)$ ?
  - c. Find an invariance between any two ordered pairs  $(p, q)$  and  $(x, y)$ , where both belong to  $S$ .
4. a) Start with a point  $S(a, b)$  of the plane with  $0 < a < b$ , we generate a sequence  $(x_n, y_n)$  of the points according to the rule

$$x_0 = a, y_0 = b, x_{n+1} = \sqrt{x_n y_{n+1}}, y_{n+1} = \sqrt{x_n y_n}$$

Prove that there is a limiting point with  $x = y$ . Find this limit.

b) Take above definition of  $S(a, b)$  with:

$$x_0 = a, y_0 = b, x_{n+1} = \frac{2x_n y_n}{x_n + y_n}, y_{n+1} = \frac{2x_{n+1} y_n}{x_{n+1} + y_n}$$

Prove that there is a limiting point with  $x = y$ . Find this limit.

5. You have six integers  $a_1, a_2, a_3, a_4, a_5, a_6$  arranged in the clock-wise fashion on a circle. Their initial values are 1, 0, 1, 0, 0, 0 respectively. You then run a loop, each iteration of which takes two consecutive integers (that is,  $(a_1, a_2)$  or  $(a_2, a_3)$  or  $\dots$  or  $(a_6, a_1)$ ), and increments both the chosen integers by 1. Your goal is to make all the six integers equal. Propose a way to achieve this using the above loop (that is, specify which pairs you choose in different iterations), or prove that this cannot be done.
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