CS21201: Discrete Structures

Autumn 2024
Tutorial 2: Logic

Problem 1

While walking in a labyrinth, you find yourself in front of three possible roads. The road on your left is paved with gold, the road in front of you is paved with marble, while the road on your right is made of small stones. Each road is protected by a guard. You talk to the guards, and this is what they tell you.

- The guard of the gold road: *"This road will bring you straight to the center. Moreover, if the stones take you to the center, then also the marble takes you to the center."*
- The guard of the marble road: *"Neither the gold nor the stones will take you to the center."*
- The guard of the stone road: *"Follow the gold, and you will reach the center. Follow the marble, and you will be lost."*

You know that all the guards are liars. Your goal is to choose a correct road that will lead you to the center of the labyrinth. Solve your problem using a propositional-logic.

Problem 2

Your task is to (logically) solve a murder mystery on behalf of Sherlock Holmes, which appeared in the novel "A Study in Scarlet" by Sir Arthur Conan Doyle. The arguments (simplified from the novel) go as follows.

1. There was a murder. If it was not done for robbery, then either it was a political assassination, or it might be for a woman.
2. In case of robbery, usually something is taken.
3. However, nothing was taken from the murderer's place.
4. Political assassins leave the place immediately after their assassination work gets completed.
5. On the contrary, the assassin left his/her tracks all over the murderer's place.
6. For an assassin, to leave tracks all over the murderer's place indicates that (s)he was there all the time (for long duration).

Logically deduce the reason for the murder.

Problem 3

Prove the following two logical deductions.

(a)

$(\neg p \lor q) \rightarrow r$
$r \rightarrow (s \lor t)$
$\neg(s \lor u)$
$t \rightarrow u$
$q \leftrightarrow v$
$(v \land \neg w) \lor (\neg v \land w) \rightarrow \neg p$

$\therefore \neg w$

(b)

$t \rightarrow q$
$\neg r \rightarrow \neg s$
$p \rightarrow u$
$\neg t \rightarrow \neg r$
$u \rightarrow s$

$\therefore p \rightarrow q$

Problem 4

(a) Given the predicate Person(x), which states that x is a person, and Ruler(x), which states that x is a ruler, write a statement in first-order logic that says “at most one person is a ruler.”

(b) Given the predicate Instant(i), which states that i is an instant in time, and Precedes(x, y), which states that x precedes y, write a sentence in first-order logic that says “time has a beginning, but has no end.”

Problem 5

Let p(x) be an open statement. Encode the following as quantified expressions.

(a) p(x) is true for exactly one value of x.

(b) p(x) is true for exactly two values of x.

(c) p(x) is true for at most one value of x.

(d) p(x) is true for at least two values of x.

(e) If p(x) is true for atleast two values of x, then p(x) is true for all values of x.

(f) If p(x) is true for atleast two values of x, then p(x) is true for all values of x.

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Question 1

Someone ate Ronaldo's cookies from his seat in the pavilion. There are three suspects: Messi, Mbappe and Haaland. Ofcourse, all three of them deny stealing. When forced to open their mouths, they say:
Messi: "Ronaldo knows Mbappe, but Haaland liked his cookies."
Mbappe: "I don't even know Ronaldo. Besides, I was on the pitch at that time."
Haaland: "I saw both Messi and Mbappe without Ronaldo in the pavilion at that time, one of them must have done it."

Assume that the two innocent men are telling the truth, but the guilty man may or may not be. Who ate the cookies? Use propositional logic.

Question 2

Universal Gates: A universal gate is a boolean logic function that can be used to realize all possible boolean logic functions. NAND and NOR are the two universal gates with their logic as defined below:

$$\text{NAND}(P,Q) = \neg(P \land Q)$$
$$\text{NOR}(P,Q) = \neg(P \lor Q)$$

Realize the following boolean logic functions using only NAND functions, you may not use the negation ($\neg$) operator:

a. $P \rightarrow Q$
b. $P \leftrightarrow Q$
c. $P \oplus Q$

Question 3

Establish the validity of the following statements using the resolution rule (alongwith the rules of inference and the laws of logic):

a.
$$\begin{array}{l} p \lor (q \land r) \\ p \rightarrow s \\ \hline \therefore\ r \lor s \end{array}$$

b.
$$\begin{array}{l} \neg p \lor s \\ \neg t \lor (s \land r) \\ \neg q \lor r \\ p \lor q \lor t \\ \hline \therefore\ r \lor s \end{array}$$

c. $\neg j \lor k$
$k \lor l$
$(l \land \neg j) \rightarrow (m \land \neg j)$
$\neg k$
___
$\therefore\ m$

d. $(\neg m \land \neg n) \rightarrow (o \rightarrow n)$
$n \rightarrow m$
$\neg m$
___
$\therefore\ \neg o$

e. $w \rightarrow x$
$(w \land x) \rightarrow y$
$(w \land y) \rightarrow z$
___
$\therefore\ w \rightarrow z$

f. $p \rightarrow (q \rightarrow r)$
$p \lor s$
$t \rightarrow q$
$\neg s$
$t \lor u$
___
$\therefore\ r \lor u$

Question 4

As a student of Discrete Structures (DS), consider the following deduction problem to be coded and solved in propositional logic. Use the following propositions specified in the table below:

Proposition	Meaning
S	TRUE, if you study DS. FALSE otherwise
D	TRUE, if you do well in the DS exam. FALSE otherwise
R	TRUE, if you relax in DS classes. FALSE otherwise
E	TRUE, if you enjoy your third semester. FALSE otherwise

a. Write the following sentences in propositional logic WITHOUT using the implication ($\rightarrow$) operator. You may use any other operator:

S1: If you study DS, then you do well in the DS exam
S2: If you relax in DS classes, then you enjoy your third semester
S3: You either study in DS classes or relax in DS classes, but not both
S4: If you study DS, then you don't enjoy your third semester
S5: If you relax in DS classes, then you don't do well in the DS exam
G: You enjoy your third semester if and only if you do not do well in the DS exam

b. Deduce G from $S1 \wedge S2 \wedge S3 \wedge S4 \wedge S5$ using Truth Tables.

Question 5

Show that each of the following arguments is invalid by providing a counterexample – an assignment of truth values of primitive statements p, q, r and s such that all premises are true (have truth value 1) while the conclusion is false (has truth value 0)

(a) $[(p \wedge \neg q) \wedge [p \to (q \to r)]] \to \neg r$

(b) $[[(p \wedge q) \to r] \wedge (\neg q \vee r)] \to p$

(c)

$$\begin{array}{l} p \leftrightarrow q \\ q \to r \\ r \vee \neg s \\ \neg s \to \neg q \\ \hline \therefore\ s \end{array}$$

Question 6

Solve using propositional logic.

(a) If Jonas goes to the meeting, then a complete report will be made; but if Jonas does not go to the meeting, then a special election will be required. If a complete report is made, then an investigation will be launched. If Jonas's going to the meeting implies that a complete report will be made, and the making of a complete report implies that an investigation will be launched, then either Jonas goes to the meeting and an investigation is launched or Jonas does not go to the meeting and no investigation is launched. If Jonas goes to the meeting and an investigation is launched, then some members will have to stand trial. But if Jonas does not go to the meeting and no investigation is launched, then the organization will disintegrate very rapidly. Therefore either some members will have to stand trial or the organization will disintegrate very rapidly.

(b) If Mr. Smith is the manager’s next-door neighbor, then Mr. Smith lives halfway between Detroit and Chicago. If Mr. Smith lives halfway between Detroit and Chicago, then he does not live in Chicago. Mr. Smith is the manager’s next-door neighbor. If Mr. Robinson lives in Detroit, then he does not live in Chicago. Mr. Robinson lives in Detroit. Mr. Smith lives in Chicago or else either Mr. Robinson or Mr. Jones lives in Chicago. If Mr. Jones lives in Chicago, then the manager is Jones. Therefore the manager is Jones.

Question 7

Consider the following statements:
C1: All lions are fierce creatures
C2: Some lions do not drink coffee
Indicate the truth of the following statements with appropriate proof/counterexample. You should use Predicate Logic with the predicates Lion(x), Fierce(x) and DrinksC(x).
a. It follows from C1 that there is a fierce creature
b. It follows from C1 and C2 that there is a fierce creature
c. It follows from C1 and C2, that some fierce creatures do not drink coffee.

Question 8

Solve the following problems using Predicate Logic by using predicates of your choice:

(a) No one respects a person who doesn't respect himself. No one will hire a person he does not respect. Therefore, a person who respects no one will never be hired by anyone.

(b) Tony, Mike and John belong to the Alpine Club. Every member of the Alpine Club is either a skier, or a mountain climber or both. No mountain climber likes rain, and all skiers like snow. Mike dislikes whatever Tony likes and likes whatever Tony dislikes. Tony likes rain and snow. Is there a member of the Alpine Club who is a mountain climber but not a skier? If so, who?