# CS21201 Discrete Structures
Tutorial Problems
Elementary Counting Techniques

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1. You are given r red balls, g green balls, and b blue balls. Assume that r, g, b are positive integers. Your task is to arrange the balls on a line subject to the following conditions. Find the count of all possible arrangements in each case.
   (a) All blue balls appear together.
   (b) No two blue balls appear together.
   (c) No blue ball can appear after any green ball.
   (d) The arrangement must start with a blue ball and end with a non-blue ball.

2. How many binary strings of length n are there with exactly k occurrences of the pattern 10? Assume that n >= 2k. [Submit]

3. How many sorted arrays of size n are there if each element of the array is an integer in the range 1,2,3,...,r?

4. Consider paths in the grid from (0, 0) to (n, n) consisting of right and up movements only. Such a path is called diagonal-crossing if it crosses the main diagonal y = x at least once. For example, the adjacent figure shows a path (for n = 4) that crosses the diagonal twice (horizontally at (2, 2) and vertically at (3, 3)). Prove that the total number of diagonal-crossing paths from (0, 0) to (n, n) is (n − 1)C(n), where C(n) is the n-th Catalan number. (Hint: First figure out the count of paths that are not diagonal-crossing.) [Submit]
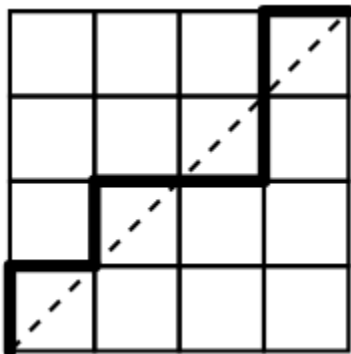
5. The tree data structure is a specialized data structure to store data in a hierarchical manner. It is used to organize and store data in the computer to be used more effectively. The tree data structure has roots, branches, and leaves connected. A full binary tree is a special kind of binary tree that has either zero or two child nodes for each node. What is the total number of full binary trees with n leaves? (Suppose the tree is unlabelled)