CS21201: Discrete Structures

Autumn 2024

Practice: Set Sizes

1. Prove that the set $A = \{a + ib \mid a, b \in Z\}$ of Gaussian Integers is countable.

2. Prove that the set of all permutations of N is not countable.

3. Consider the set $S = \{a + b\sqrt{7} \mid a, b \in Z\}$. Prove that $R - S$ is uncountable.

4. Provide an explicit bijection between N and N×N. It should not be an exhaustive enumeration.

5. Determine whether the following sets are countable or uncountable
   (a) The set of all finite subsets of N
   (b) The set of all infinite subsets of N

6. An infinite bit sequence is an infinite sequence of 0s and 1s. Denote S as the set of all infinite bit sequences. Let $\alpha(n)$ be the n-th element of an infinite bit sequence $\alpha \in S$. Determine whether the following sets are countable or uncountable:
   (a) S
   (b) $T = \{\alpha \in S \mid \alpha(n) = 1 \text{ and } \alpha(n + 1) = 0 \text{ for } \underline{\text{some}}\ n \geq 0\}$
   (c) $T = \{\alpha \in S \mid \alpha(n) = 1 \text{ and } \alpha(n + 1) = 0 \text{ for } \underline{\text{no}}\ n \geq 0\}$

7. We have the following sets, determine whether they are countable or uncountable.
   (a) The set of all functions from N to {1, 2}
   (b) The set of all functions from N to N
   (c) The set of all functions from {1, 2} to N
   (d) The set of all non-increasing functions from N to N