CS21201 Discrete Structures

Practice Problems

Abstract Algebraic Structures

1. Define two operations on $\mathbb{Z}$ as

$$a \oplus b = a + b + u,$$
$$a \odot b = a + b + vab,$$

where u, v are constant integers. For which values of u and v, is $(\mathbb{Z}, \oplus, \odot)$ a ring?

2. Take $u = v = 1$ is Exercise 1.

(a) Find the units of $(\mathbb{Z}, \oplus, \odot)$. Find their respective inverses.

(b) Prove that the set of all odd integers is a subring of this ring. What about the set of all even integers?

3. Let $\mathbb{Z}_1$ be the ring of Exercise 1 with $u = v = 1$, and $\mathbb{Z}_2$ the ring of Exercise 1 with $u = v = -1$. Define a ring isomorphism $\mathbb{Z}_1 \to \mathbb{Z}_2$.

4.

Let K, L be fields, and $f : K \to L$ a non-zero ring homomorphism.

(a) Prove/disprove: $f(1_K) = 1_L$.

(b) Prove that f is injective.

5.

What is the inverse of an element a in the group $G = \{a \in \mathbb{R} \mid a > 0\}$ under the operation $\odot$ defined by $a \odot b = a^{\ln b}$?

6. Let (R,+,·) be a ring such that for every x ∈ R, x·x = x. Prove or disprove that R is a commutative

7. Let A,B are subgroups of a group G. Prove or disprove that A∩B is also a subgroup of G.