Counter-Example Guided Abstraction Refinement

Testing & Verification

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Reference

This presentation is based on the work of E. Clarke, A. Gupta, J. Kukula, O. Strichman (CAV'02). Most of these slides are from A. Gupta's presentation.

Model Checking

- **Given:**
  - Finite transition system $M(S, I, R, L)$
  - A temporal property p

- **The model checking problem:**
  - Does M satisfy p?

$$M \overset{?}{\models} p$$

Model Checking

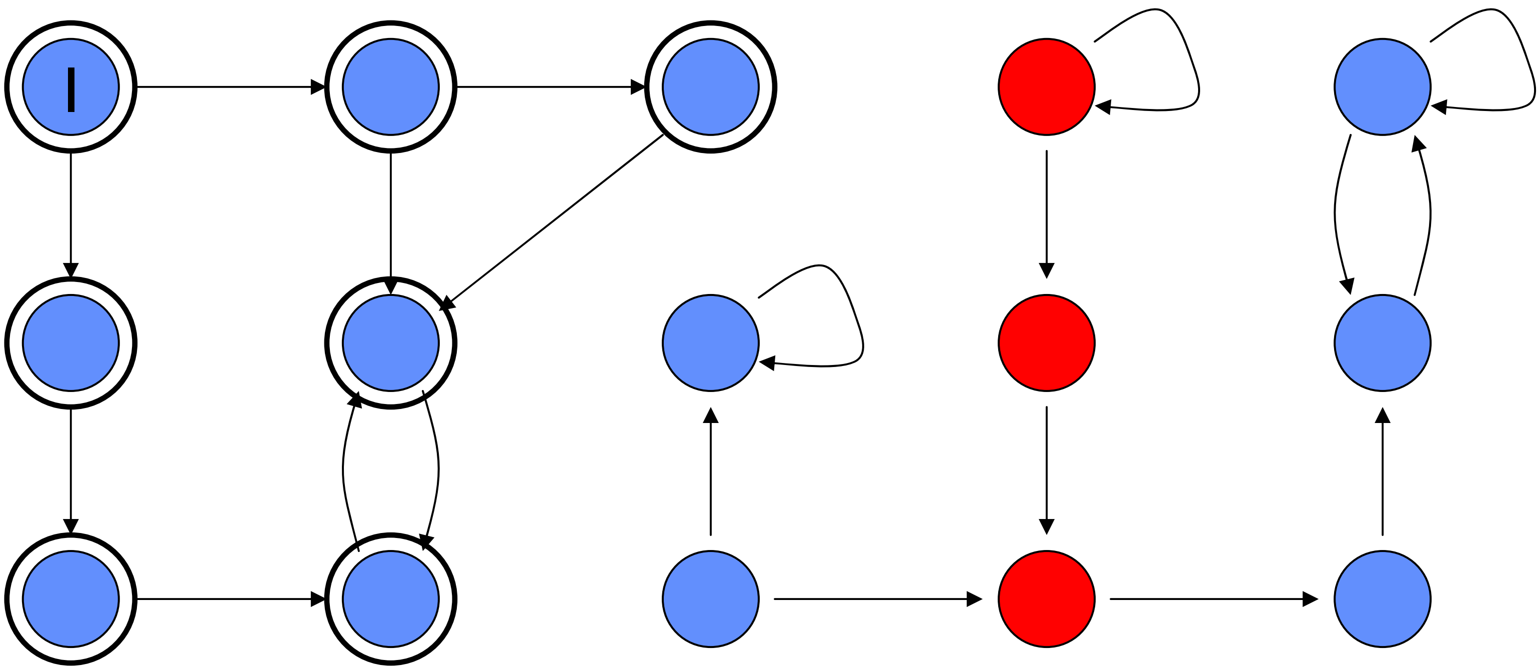
- **Temporal properties:**
  - **"Always x=y"**
    (G(x=y))
  - **"Every Send is followed immediately by Ack"**
    (G(Send → X Ack))

  "Safety" properties

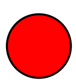
  - **"Reset can always be reached"**
    (GF Reset)
  - **"From some point on, always switch_on"**
    (FG switch_on)

  "Liveness" properties

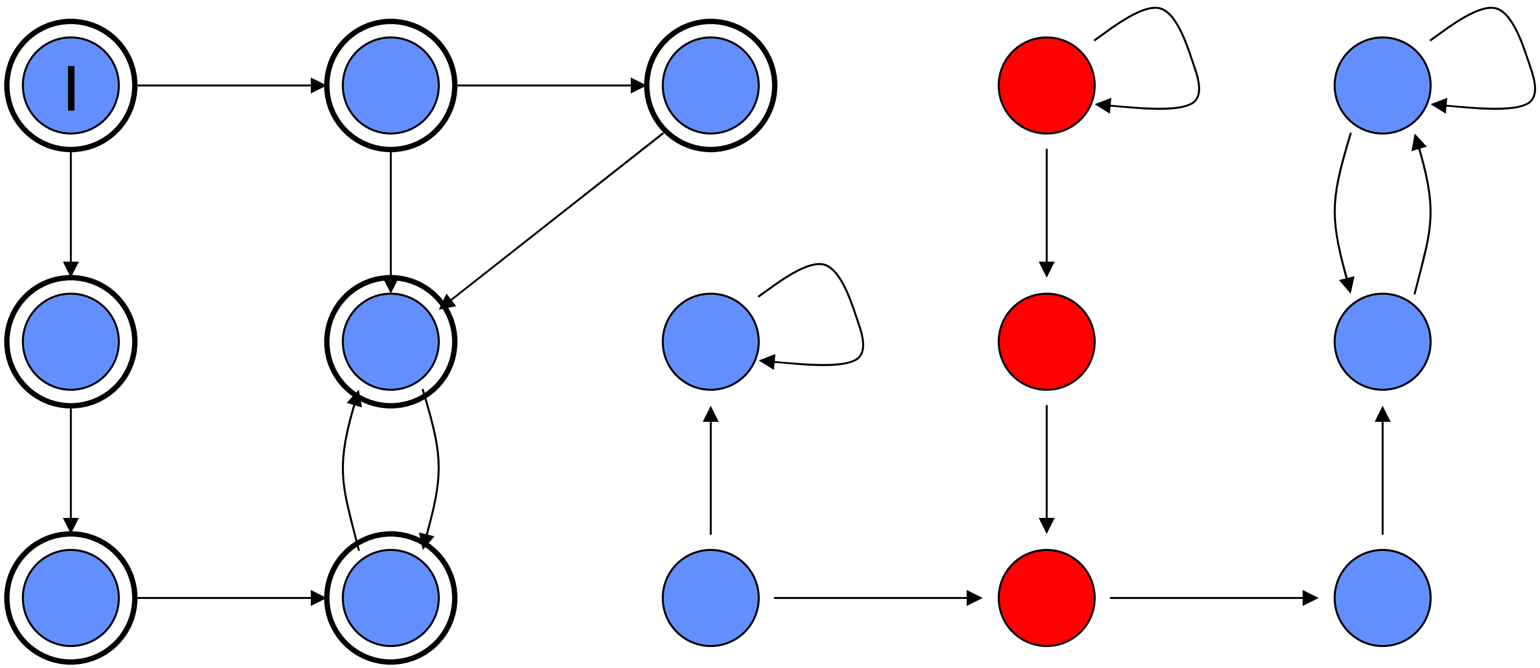
Model Checking (safety)



Add reachable states until reaching a fixed-point

 = bad state

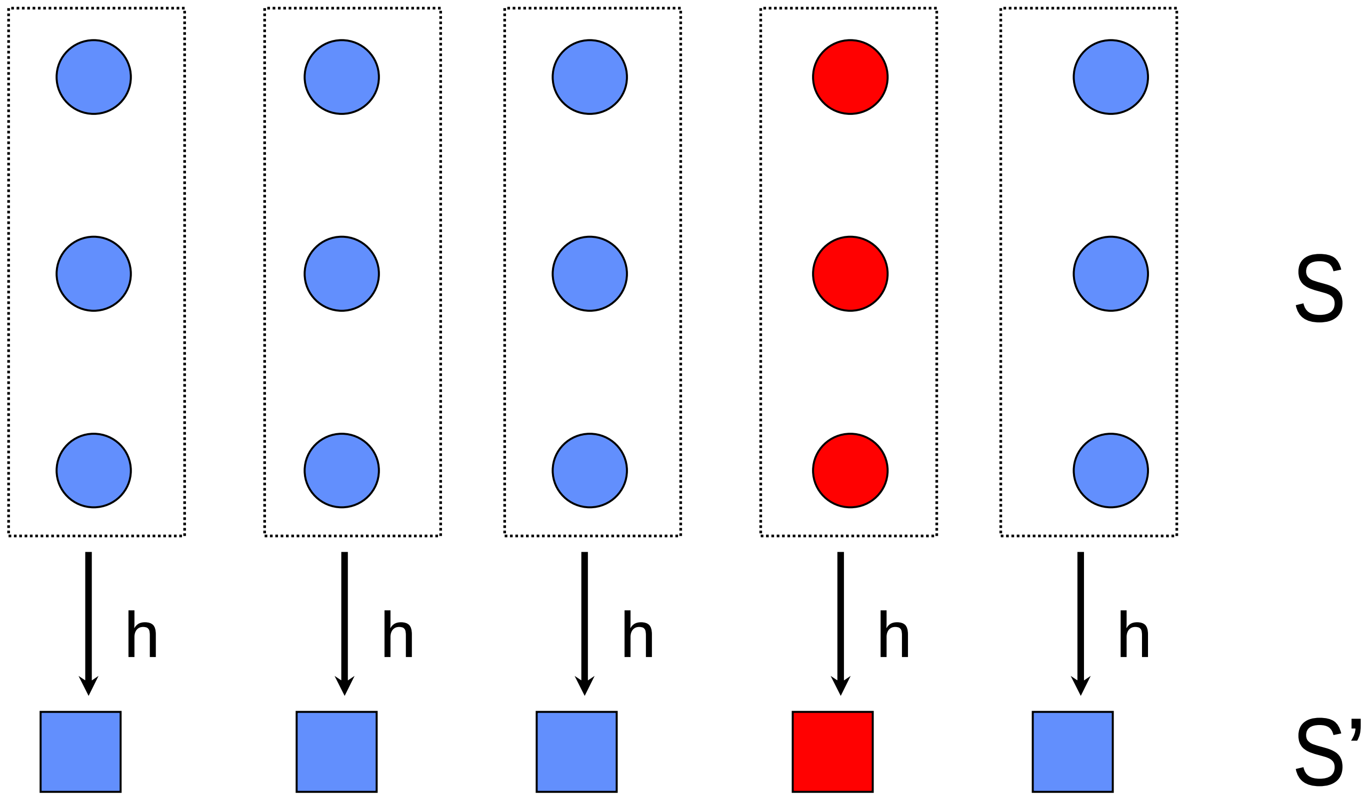
Model Checking (safety)



Too many states to handle !

= bad state

Abstraction



Abstraction Function h : S ! S’

Abstraction Function

- Partition variables into visible(V) and invisible(I) variables.

  - The abstract model consists of V variables. I variables are made inputs.

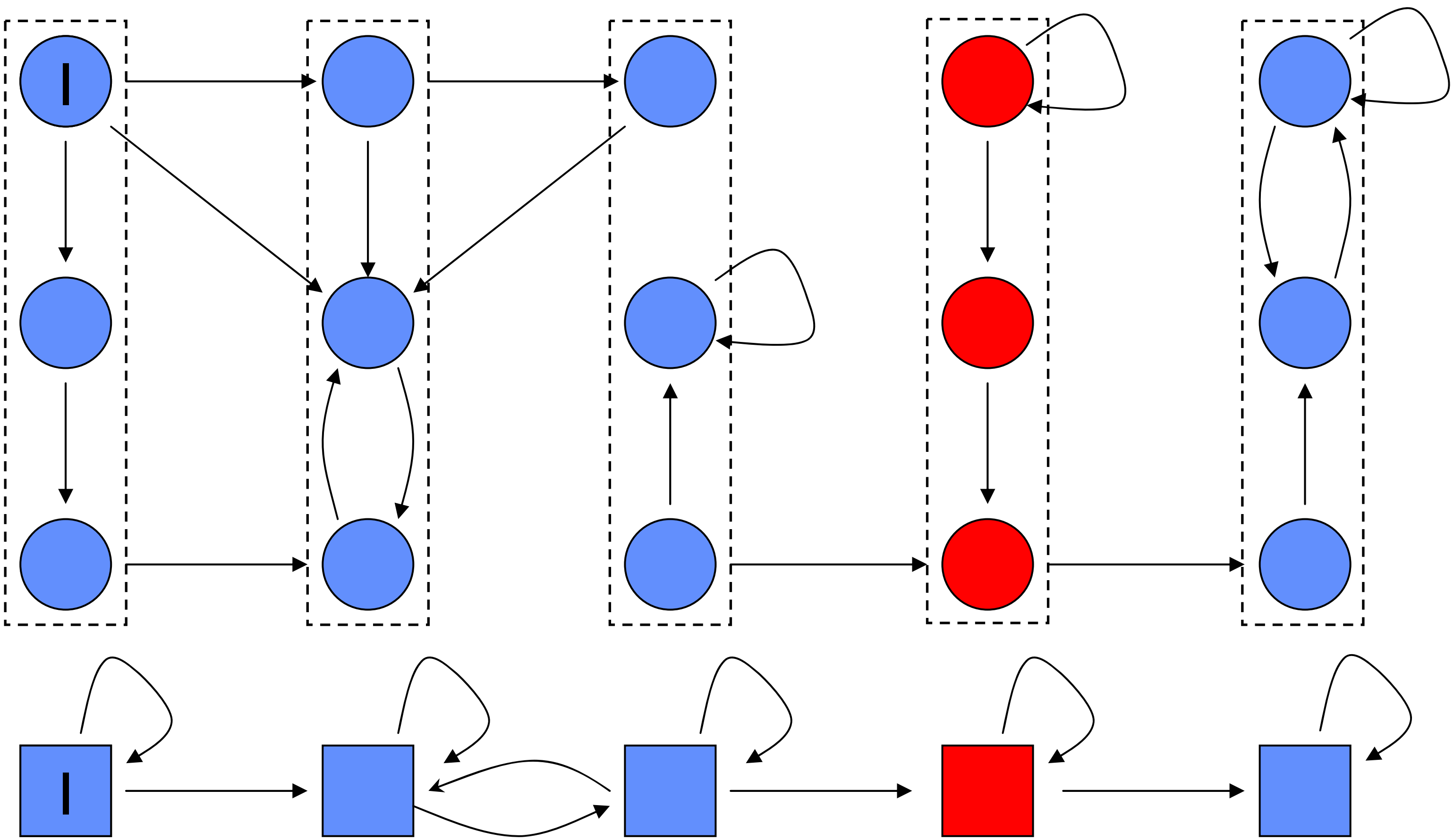
  - The abstraction function maps each state to its projection over V.

Abstraction Function

$$\begin{array}{cccc} x1 & x2 & x3 & x4 \\ \hline 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{array} \xrightarrow{\ \mathrm{h}\ } \begin{array}{cc} x1 & x2 \\ \hline 0 & 0 \end{array}$$

Group concrete states with identical visible part to a single abstract state.

Existential Abstraction



Model Checking Abstract Model

- **Preservation Theorem**

$$M' \models p \rightarrow M \models p$$

- **Converse does not hold**

$$M' \not\models p \not\rightarrow M \not\models p$$

- **The counterexample may be spurious**

Checking the Counterexample

- Counterexample : $(c_1, \ldots, c_m)$
  - Each c_i is an assignment to V.

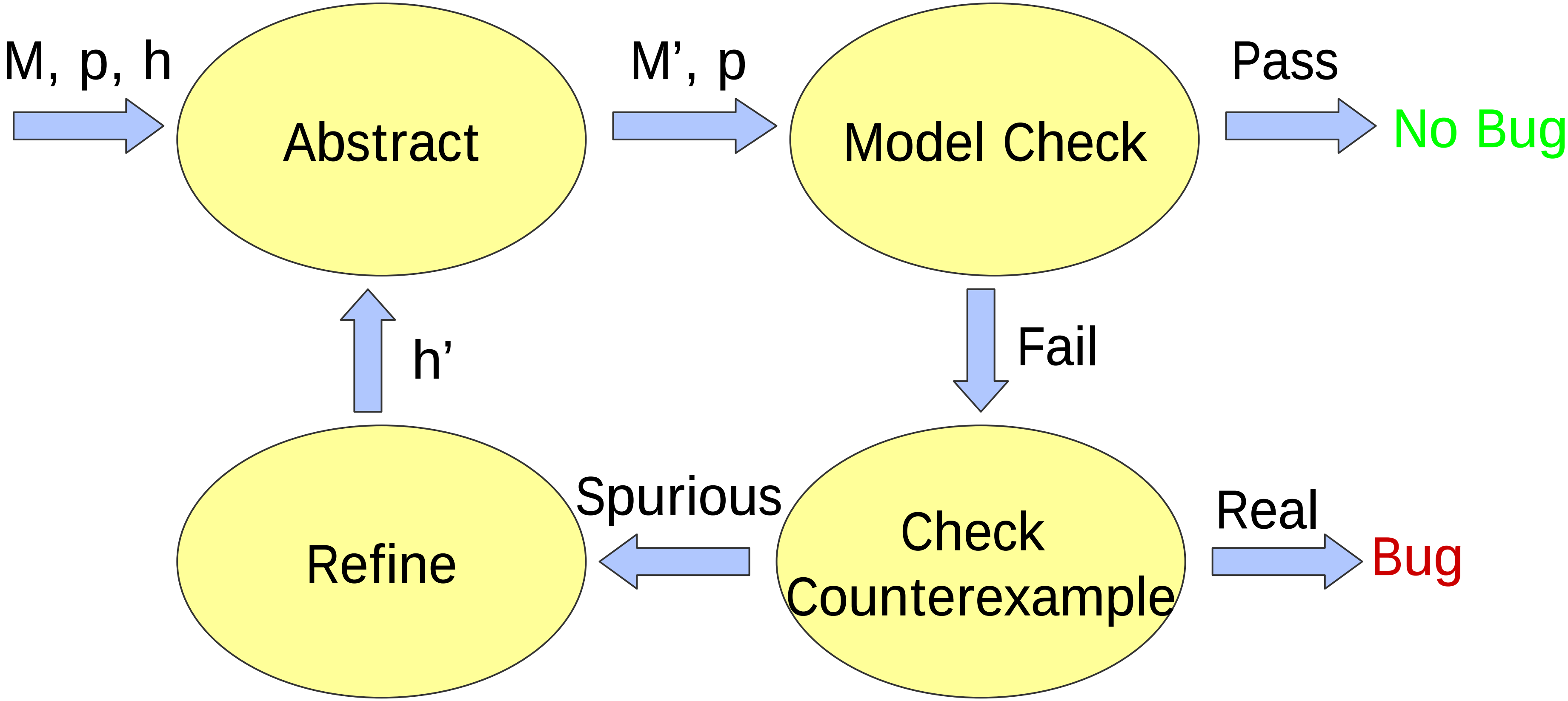
- Simulate the counterexample on the concrete model.

Checking the Counterexample

Concrete traces corresponding to the counterexample:

$\phi \;=\; I(s_1) \;\wedge$	**(Initial State)**
$\bigwedge_{i=1}^{m-1} R(s_i, s_{i+1}) \;\wedge$	**(Unrolled Transition Relation)**
$\bigwedge_{i=1}^{m} \mathsf{visible}(s_i) = c_i$	**(Restriction of V to Counterexample)**

Abstraction-Refinement Loop



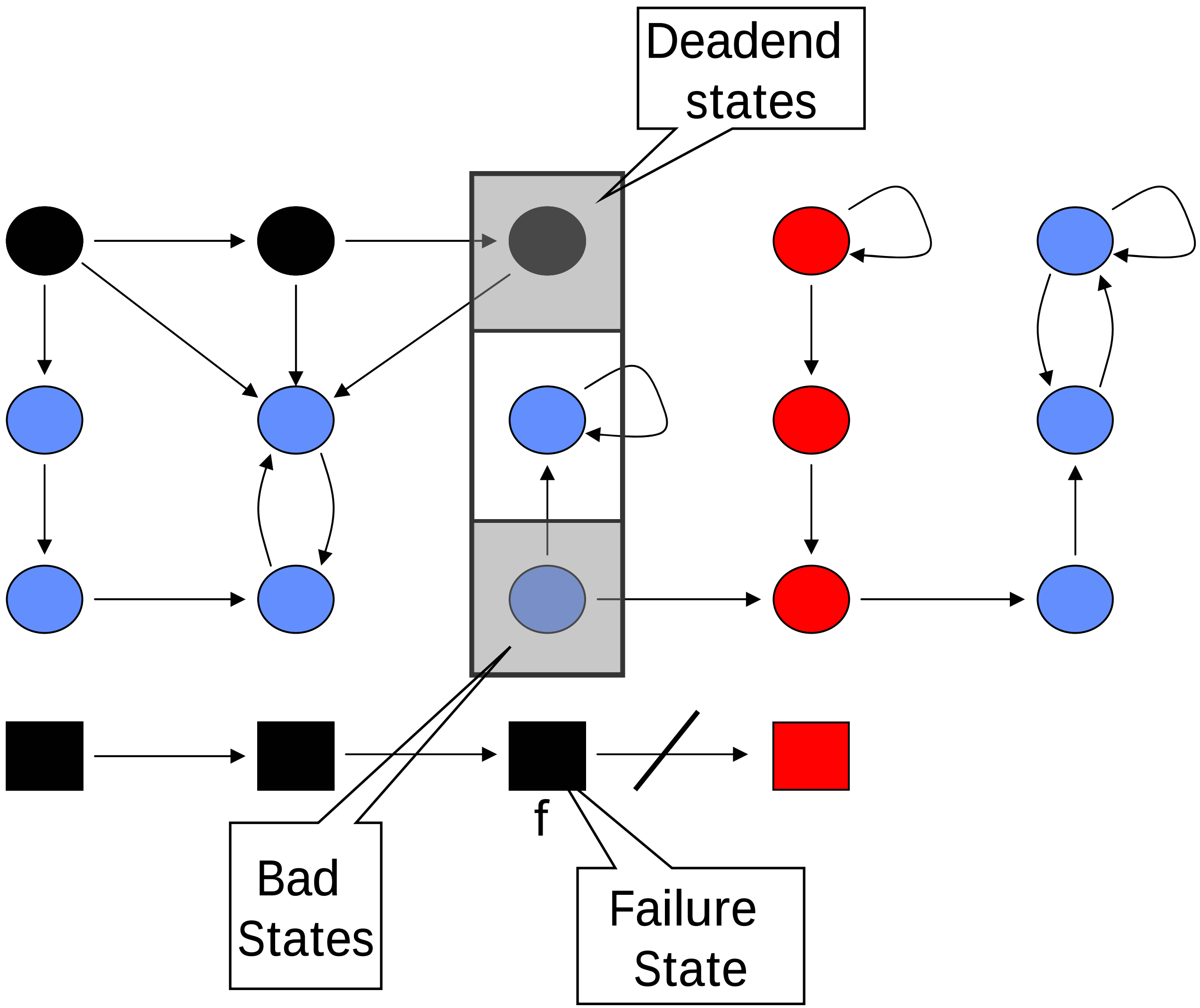
Refinement

Abstraction/refinement with conflict analysis

(Chauhan, Clarke, Kukula, Sapra, Veith, Wang, FMCAD 2002)

- **Simulate counterexample on concrete model with SAT**
- **If the instance is unsatisfiable, analyze conflict**
- **Make visible one of the variables in the clauses that lead to the conflict**

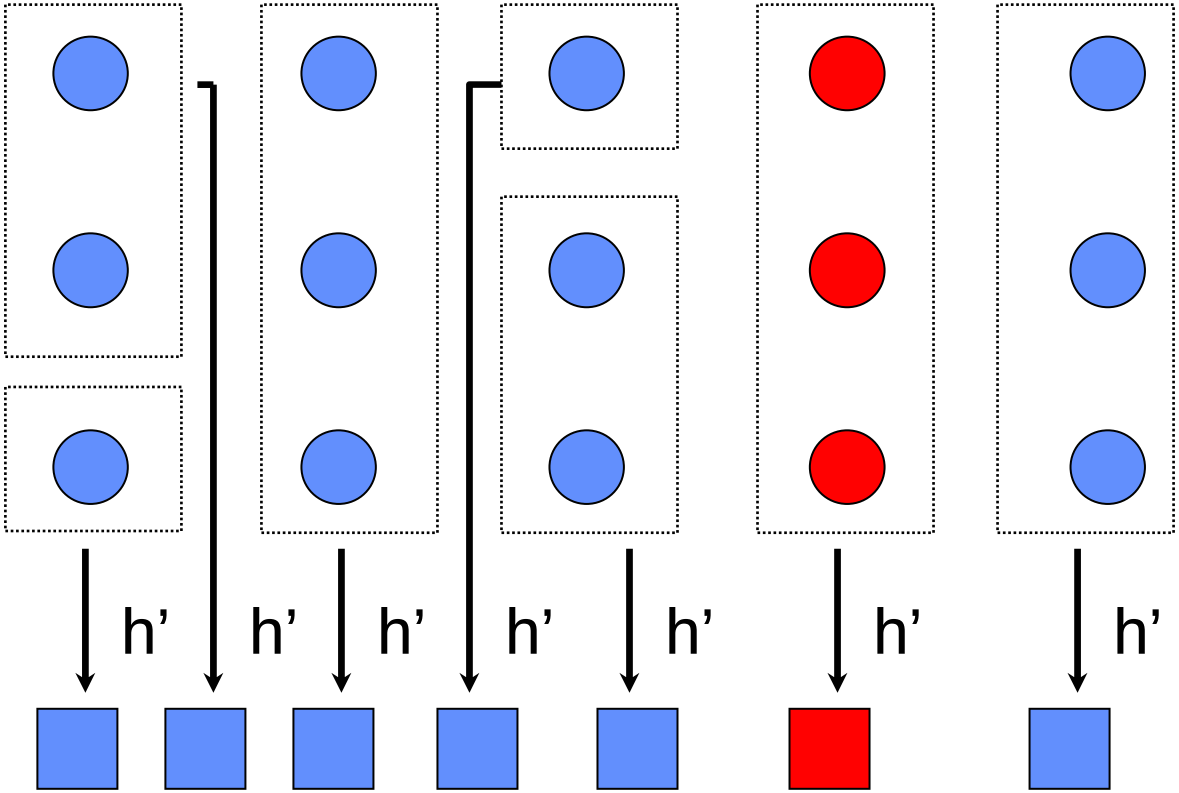
Why spurious counterexample?



Refinement

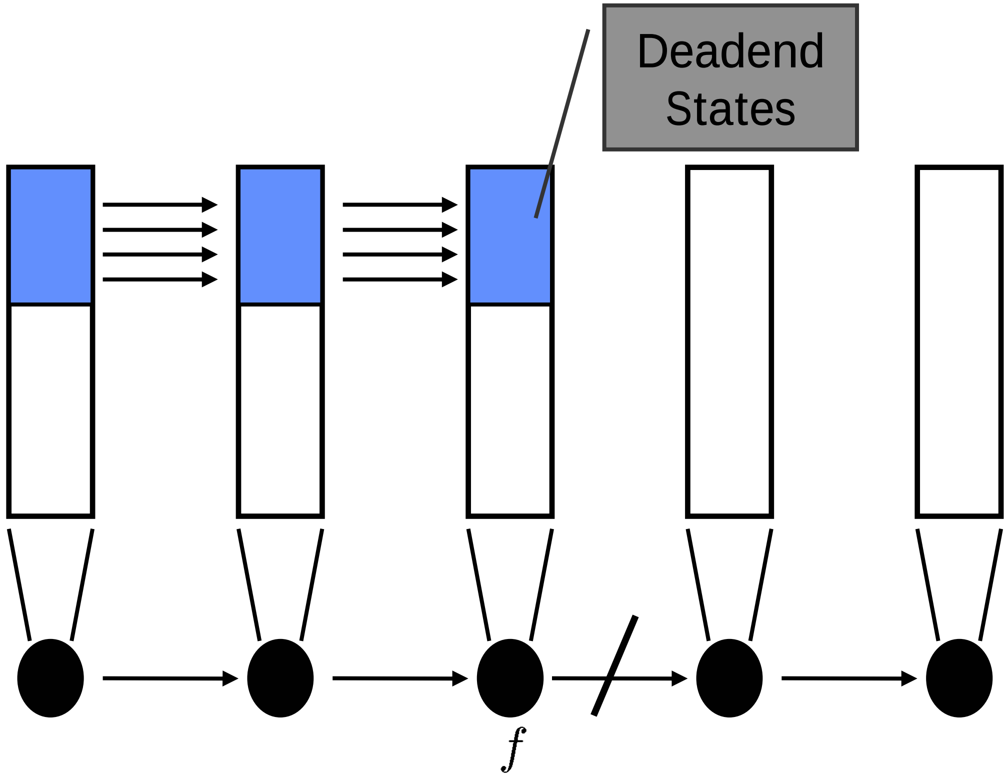
- Problem: Deadend and Bad States are in the same abstract state.
- Solution: Refine abstraction function.
- The sets of Deadend and Bad states should be separated into different abstract states.

Refinement



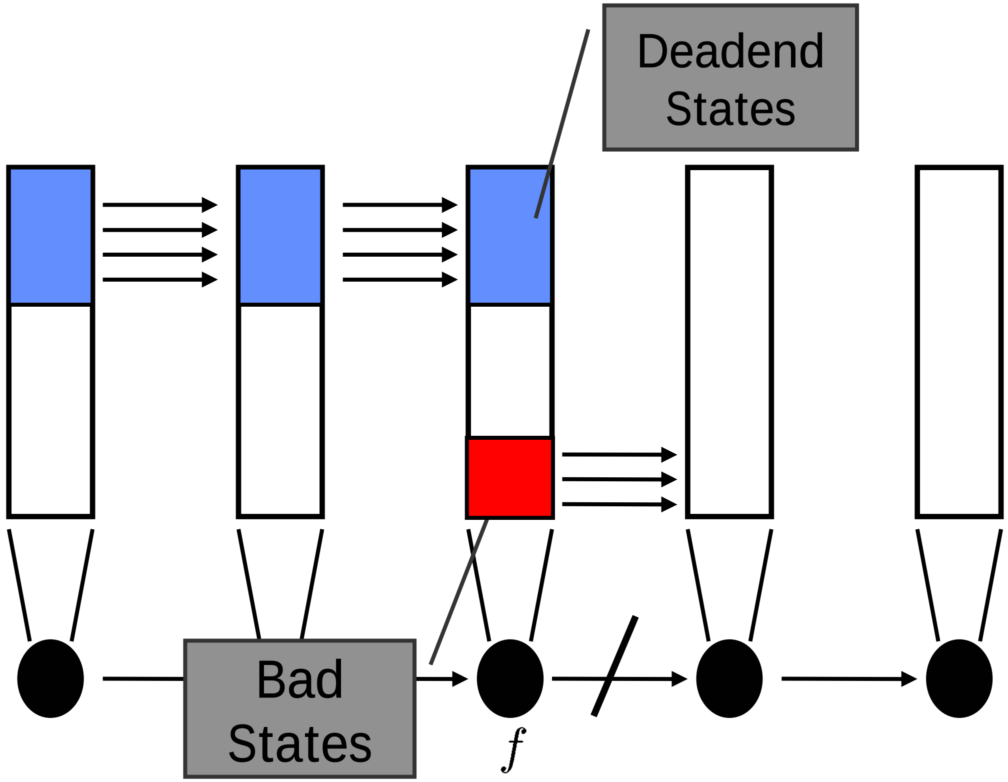
Refinement : h’

Refinement



$$\phi_D = I(s_1) \wedge \bigwedge_{i=1}^{f-1} R(s_i, s_{i+1}) \wedge \bigwedge_{i=1}^{f} \text{visible}(s_i) = c_i$$

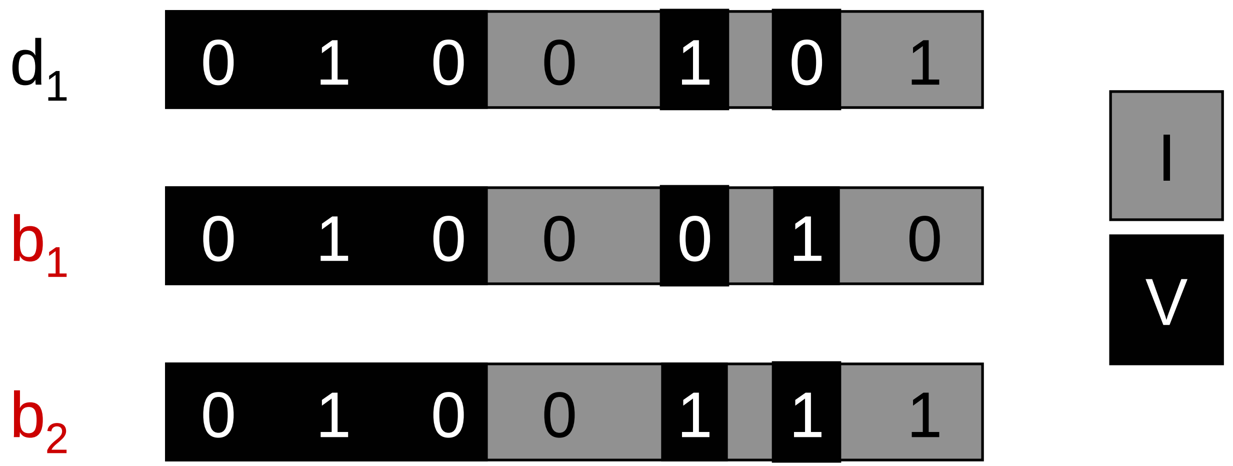
Refinement



$$\phi_B = R(s_f, s_{f+1}) \ \wedge$$

$$\text{visible}(s_f) = c_f \wedge \text{visible}(s_{f+1}) = c_{f+1}$$

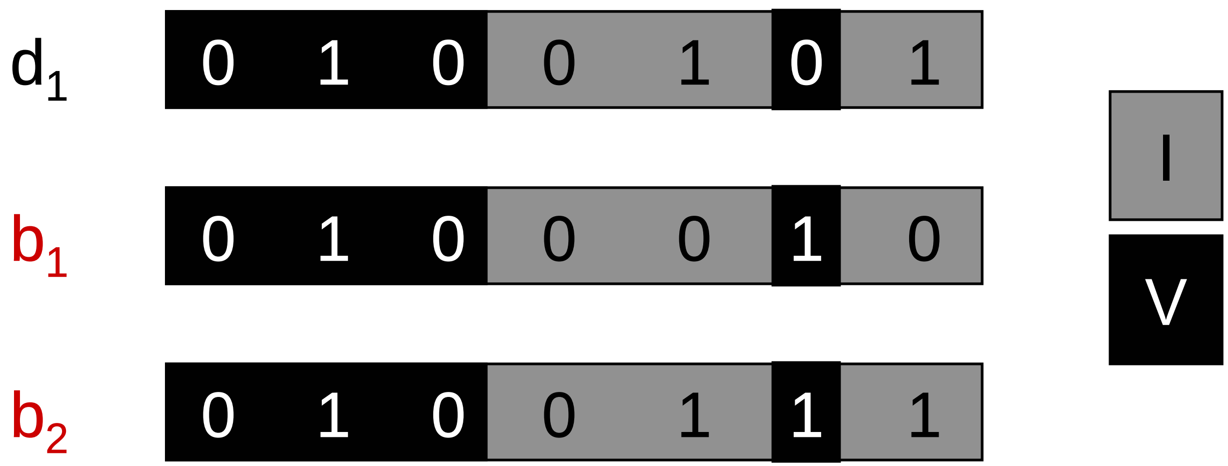
Refinement as Separation



Refinement : Find subset U of I that separates between all pairs of deadend and bad states. Make them visible.

Keep U small !

Refinement as Separation



Refinement : Find subset U of I that separates between all pairs of deadend and bad states. Make them visible.

Keep U small !

Refinement as Separation

The state separation problem

Input: Sets D, B

Output: Minimal U $\in$ I s.t.:

$$\forall\, d \in D,\ \forall\, b \in B,\ \exists u \in U.\ \ d(u) \neq b(u)$$

The refinement h' is obtained by adding U to V.

Two separation methods

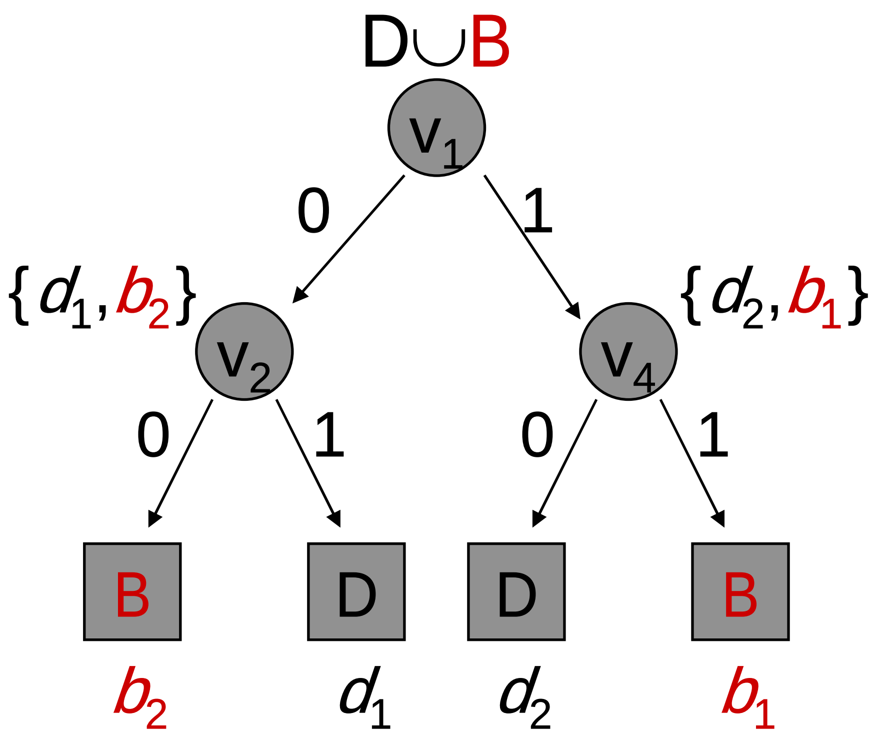
- **ILP-based separation**
  - Minimal separating set.
  - Computationally expensive.

- **Decision Tree Learning based separation.**
  - Not optimal.
  - Polynomial.

Separation with Decision Tree Learning (Example)

Classification:

D	B
$d_1 = (0,1,0,1)$	$b_1 = (1,1,1,1)$
$d_2 = (1,1,1,0)$	$b_2 = (0,0,0,1)$



Separating Set : $\{v_1, v_2, v_4\}$

Separation with 0-1 ILP (Example)

$$d_1 = (0,1,0,1) \qquad b_1 = (1,1,1,1)$$
$$d_2 = (1,1,1,0) \qquad b_2 = (0,0,0,1)$$

Min $\sum_{i=1}^{4} v_i$

subject to:

$$\begin{array}{lll} v_1 + v_3 & \geq 1 & /* \text{ Separating } d_1 \text{ from } b_1 */ \\ v_2 & \geq 1 & /* \text{ Separating } d_1 \text{ from } b_2 */ \\ v_4 & \geq 1 & /* \text{ Separating } d_2 \text{ from } b_1 */ \\ v_1 + v_2 + v_3 + v_4 & \geq 1 & /* \text{ Separating } d_2 \text{ from } b_2 */ \end{array}$$

Separation with 0-1 ILP

$$\text{Min } \sum_{i=1}^{|\mathcal{I}|} v_i$$

$$\text{subject to:} \quad (\forall d \in D)\ (\forall b \in B) \sum_{\substack{1 \le i \le |\mathcal{I}|, \\ d,b \text{ differ at } v_i}} v_i \ge 1$$

- One constraint per pair of states.
- $v_i = 1$ iff v_i is in the separating set.

Refinement as Learning

- **For systems of realistic size**
  - **Not possible to generate D and B.**
  - **Expensive to separate D and B.**

- **Solution:**
  - **Sample D and B**
  - **Infer separating variables from the samples.**

- **The method is still complete:**
  - **Counterexample will eventually be eliminated.**

The CMU CEGAR Tool