

1. Write a recursive function, `ipow(x, n)`, to return the value of x^n , where n is a non-negative integer, using repeated squaring, that is:

$$x^{2n} = (x^n) \times (x^n) \quad \text{and} \quad x^{2n+1} = (x) \times (x^{2n})$$

- (a) Write a main program that reads a floating point number, x , and an integer, n , and calls the function to return and print the n th power of x .
- (b) In the function use a global variable, `count`, to count the number of multiplications performed.
2. You are given a fair dice and asked to compute the probability of having k sixes in n rolls of the dice. The probability is given by the Binomial term:

$$P(k, n) = {}^n C_k p^k (1 - p)^{n-k}$$

The probability of getting a six in a single roll of the dice is $p = 1/6$. The probability of getting at most k sixes in n rolls of the dice is given by:

$$P(\leq k, n) = \sum_{j=0}^k {}^n C_j p^j (1 - p)^{n-j}$$

- (a) For computing $P(\leq k, n)$, we need the value of ${}^n C_j$ while computing the j^{th} term of the summation. We know that ${}^n C_j = \frac{n-j+1}{j} {}^n C_{j-1}$ and therefore it is easy to compute ${}^n C_j$ from ${}^n C_{j-1}$ which was anyway computed for the $(j-1)^{\text{th}}$ term. Write a function, `getterm`, which returns the value of ${}^n C_0$ when called the first time, ${}^n C_1$ when called the second time, ${}^n C_2$ when called the third time (use a static variable).
- (b) Write a function for computing $P(\leq k, n)$ using the function `getterm` and a `main()` to read the values of k and n and print the value of $P(\leq k, n)$.

3. **Polynomials.** A polynomial $a_0 + a_1x + a_2x^2 + \dots + a_kx^k$ of degree k can be represented by a single dimensional array, $A[]$, of $k+1$ floating pointing numbers, where $A[j] = a_j$. Write the following functions in C:

Function Prototype	Description
<code>void read_poly(FILE *fp, float A[], int k)</code>	Reads coefficients of a polynomial of degree k from a file into array A
<code>float eval_poly(float A[], int k, float x)</code>	Returns the value of polynomial A for given value of x
<code>void add_poly(float A[], float B[], float C[], int k)</code>	Adds polynomials A and B into C
<code>void mul_poly(float A[], float B[], float C[], int k)</code>	Multiplies polynomials A and B into C
<code>void print_poly(float A[], int k)</code>	Prints the polynomial

Write a program, **asg11.c**, which does the following:

- (a) It opens a file, `input.dat`, using the following code:

```
FILE *fp, *fopen();
fp = fopen("input.dat", "r");
if (fp == NULL) { printf("Unable to open file.\n");
                  exit(0); }
```

- (b) It reads the value of k (assume that it is always less than 10) from the file.
(c) It uses the function `read_poly()` to read polynomials A and B of degree k from the file.
(d) It uses the function `add_poly()` to find the polynomial C representing the sum of A and B
(e) It uses the function `mul_poly()` to find polynomial D representing the product of A and B .
(f) It uses the function `print_poly()` to print the polynomials, A , B , C , and D into the terminal.
(g) It reads a value of x from the terminal.
(h) It uses `eval_poly()` to compute the values of the polynomials, A , B , C , and D . These values are then printed into the terminal.

For the polynomials, $p(x) = 3x^4 + 5x^2 - 7.5x + 20$ and $q(x) = 8x^4 + 9.2x^3 - 14$, the sample format of the input file is as follows (the first line has the value of k):

```
4
20 -7.5 5 0 3
-14 0 0 9.2 8
```

4. The ministry of magic produces coins of denomination 3, 5 and 10 respectively. The function, `canchange(k)`, returns `-1` if it is not possible to pay a value of k using these coins. Otherwise it returns the *minimum* number of coins needed to make the payment.

For example, `canchange(7)` will return `-1`. On the other hand, `canchange(14)` will return `4` because 14 can be paid as $3+3+3+5$ and there is no other way to pay with fewer coins.

A code skeleton for the function is given below as a hint. This is not complete, and has missing statements and missing expressions, indicated with question marks.

```
int canchange(int k)
{
    int a= ?? ;
    if (k==0) return 0;
    if ( ?? ) return 1;
    if (k < 3) ??;

    a = canchange( ?? );
    if (a > 0) return ?? ;

    a = canchange(k - 5);
    if (a > 0) return ?? ;

    a = canchange( ?? );
    if (a > 0) return ?? ;
    ??
}
```

- (a) Complete the function and write a `main()` to read an input number, call the function with it, and print the value it returns.
- (b) Modify the function of part (a) to write a function to print the change. For example, if we call the function `printchange(14)` it should print $3+3+3+5$. The function prototype is:

```
int printchange(int k)
```