1. Write LTL properties over the alphabet $\mathrm{A}, \mathrm{B}, \mathrm{C}$ for each of the following statements.
(a) $\mathrm{G}[\mathrm{A} \Longrightarrow \mathrm{X}((\mathrm{B} \mathrm{UC}) \vee \mathrm{GB})]$
(b) $\mathrm{G}[\mathrm{A} \Longrightarrow \mathrm{X}((\neg \mathrm{A} U \mathrm{~B}) \vee \mathrm{G}(\neg \mathrm{A}))]$
(c) $\mathrm{G}[\mathrm{XA} \wedge \mathrm{XXB} \Longrightarrow \mathrm{C}]$
(d) $\mathrm{G}[\mathrm{A} \wedge \mathrm{XB} \Longrightarrow \mathrm{XXC} \vee \mathrm{XXXC}] \vee$
$G[A \wedge X X B \Longrightarrow X X X C \vee X X X X C] \vee$
$\mathrm{G}[\mathrm{A} \wedge \mathrm{XXXB} \Longrightarrow \mathrm{XXXXC} \vee \mathrm{XXXXXXC}]$
(e) $\mathrm{G}[\mathrm{A} \Longrightarrow \neg \mathrm{XFB} \vee \mathrm{X}(\neg \mathrm{B} \mathrm{U}(\mathrm{B} \wedge \mathrm{FC}))]$
2. Draw the ROBDD for $f$ using the ordering $a>b>c>d$.

The given circuit reduces to $\bar{a}+\bar{d}$. Hence the BDD is as follows.

3. Equivalence Checking
(a) Not equivalent. LHS can be satisfied even when no state satisfies $\mathrm{p} \wedge \mathrm{q}$.
(b) Equivalent
(c) Not equivalent. If start state satisfies $\neg \mathrm{p} \wedge \neg \mathrm{q} \wedge \mathrm{r}$, then RHS is satisfied, but LHS fails.
4. 3-bit Gray Counter
(a) The equations for $x_{1}^{\prime}, x_{2}^{\prime}$, and $x_{3}^{\prime \prime}$ can be written as follows

- $x_{1}^{\prime \prime}=\overline{x_{1}} x_{2} \overline{x_{3}}+x_{1} x_{2} \overline{x_{3}}+x_{1} x_{2} x_{3}+x_{1} \overline{x_{2}} x_{3}=x_{2} \overline{x_{3}}+x_{1} x_{3}$
- $x_{2}^{\prime}=\overline{x_{1}} \overline{x_{2}} x_{3}+\overline{x_{1}} x_{2} x_{3}+\overline{x_{1}} x_{2} \overline{x_{3}}+x_{1} x_{2} \overline{x_{3}}=x_{2} \overline{x_{3}}+\overline{x_{1}} x_{3}$
- $x_{3}^{\prime \prime}=\overline{x_{1}} \overline{x_{2}} \overline{x_{3}}+\overline{x_{1}} \overline{x_{2}} x_{3}+x_{1} x_{2} \overline{x_{3}}+x_{1} x_{2} x_{3}=x_{1} x_{2}+\overline{x_{1}} \overline{x_{2}}$

Therefore, $c \mathrm{f}=\left[x_{1}^{\prime} \odot\left(x_{2} \overline{x_{3}}+x_{1} x_{3}\right)\right]\left[x_{2} \prime \odot\left(x_{2} \overline{x_{3}}+\overline{x_{1}} x_{3}\right)\right]\left[x_{3}^{\prime} \odot\left(x_{1} x_{2}+\overline{x_{1}} \overline{x_{2}}\right)\right]$
(b) $\psi=\left[\left(x_{1}^{\prime}=\overline{x_{1}}\right) \Longleftrightarrow\left(x_{2}^{\prime}=x_{2}\right)\left(x_{3}^{\prime}=x_{3}\right)\right]$
$\wedge\left[\left(x_{2}^{\prime}=\bar{x}_{2}\right) \Longleftrightarrow\left(x_{1}^{\prime}=x_{1}\right)\left(x_{3}^{\prime}=x_{3}\right)\right]$
$\wedge\left[\left(x_{3}^{\prime}=\overline{x_{3}}\right) \Longleftrightarrow\left(x_{1}^{\prime}=x_{1}\right)\left(x_{2}^{\prime}=x_{2}\right)\right]$
$\therefore \varphi=\neg \psi \wedge c \mathrm{f}$
(c) $\zeta=G\left[\bigwedge_{i=2}^{i=7} \neg\left[\left(x_{1}=X^{i} x_{1}\right)\left(x_{2}=X^{i} x_{2}\right)\left(x_{3}=X^{i} x_{3}\right)\right]\right]$

