INDIAN INSTITUTE OF TECHNOLOGY KHARAGPUR

Date: 05-02-2020 Time: *60 min* Full marks: 30 No. of students: 19

Spring Semester Class Test 1, 2019/20 Dept: *Comp. Sc & Engg.* Sub No: CS60030

M.Tech (Elective) Sub Name: ***Formal Systems***

Instructions: ***Answer all questions.***

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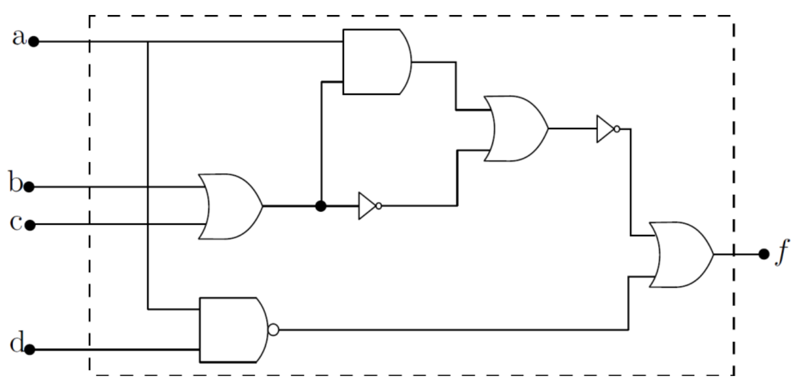
1. Write LTL properties over the alphabet {A,B,C} for each of the following statements.
   (a) An A is followed by B's forever or until C.
   (b) Between any two A's there is at least one B.
   (c) Never is it that an A is followed by a B unless the A is preceded by a C.
   (d) If an A occurs and within the next 3 symbols a B occurs, then after the B a C occurs within the next 2 symbols.
   (e) If an A occurs and is thereafter followed at some time by a B, then eventually thereafter a C occurs.

[2x5 = 10 marks]

2. Draw the ROBDD for ***f*** using the ordering a>b>c>d, for the circuit given below.



[5 marks]

3. For each of the following pairs of CTL formulas, determine whether the two formulas are equivalent. For the ones which are non-equivalent give a sample transition system where one is true and the other is false.
   (a) $(\text{AF } p) \wedge (\text{AF } q)$ and $\text{AF}(p \wedge q)$
   (b) $(\text{AG } p) \wedge (\text{AG } q)$ and $\text{AG}(p \wedge q)$
   (c) $\text{E}(p \text{ U } (q \wedge \text{E}(q \text{ U } r)))$ and $\text{E}((p \vee q) \text{ U } r)$

[2x3 = 6 marks]

4. Consider a 3-bit counter whose counting sequence is shown below.

   000 → 001 → 011 → 010 → 110 → 111 → 101 → 100 → 000 ….

   Here the state is represented by a vector $\langle x_1, x_2, x_3 \rangle$ of 3 state variables. Let $\langle x_1', x_2', x_3' \rangle$ denote the next state.

   (a) Develop the characteristic function, $cf(x_1, x_2, x_3, x_1', x_2', x_3')$, representing the transition relation of the counter. The function should be shown as a Boolean function.

   (b) We wish to determine whether the counter is a Gray counter. For this purpose we need to check from the transition relation of part (a) that successive states differ in only one bit. Prepare a Boolean formula, φ, such that the satisfiability of φ will enable you to determine whether the transition relation is one for the Gray counter.

   (c) The property of part (b) is not sufficient to establish that the transition relation is that for a 3-bit Gray counter. For example, consider the following transition relation which satisfies the property of part (b), but does not represent the transition relation of a 3-bit Gray counter.

   000 → 001 → 101 → 100 → 000

   What property do we need to add to guarantee that all 8 eight states are visited? How shall we use the characteristic function for the transition relation to prove this?

[3+3+3 = 9 marks]