Logical Deduction: I

Introduction to Propositional Logic

Partha P Chakrabarti

Indian Institute of Technology Kharagpur

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History of Logic and Deduction

Role in Artificial Intelligence and Design of Intelligent Agents

First Few Examples

- If I am the Director then I am well-known. I am the Director. So I am well-known
- If I am the Director then I am well-known. I am not the Director. So I am not well-known.
- If Rajat is the Director then Rajat is well-known. Rajat is the Director. So Rajat is well known.
- If a cat is orange coloured then the cat is a foreign cat. Pussy-Cat is blue coloured. Therefore Pussy-Cat is not a foreign cat.
- Wherever Mary goes, so does the lamb. Mary goes to school. So the lamb goes to school.

Propositional / Boolean Logic

- Boolean variables a, b, c, d, … which can take values true or false.
- Boolean formulae developed using well defined connectors ~, Λ, V, →, etc, whose meaning (semantics) is given by their truth tables.
- Codification of sentences of the argument into Boolean Formulae.
- Developing the Deduction Process as obtaining truth of a combined formula expressing the complete argument.
- Determining the Truth or Validity of the formula and thereby proving or disproving the argument and Analyzing its truth under various interpretations.

Example: 1

If I am the Director then I am well-known. I am the Director. So I am well-known

Coding: Variables

a: I am the Director

b: I am well-known

Coding the sentences:

1. $a \rightarrow b$
2. a
3. b

The final formula for deduction:

$((a \rightarrow b) \land a) \rightarrow b$

Proof or Otherwise

a	b	a → b	(a → b) Λ a	((a → b) Λ a) → b
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

Example: 2

If I am the Director then I am well-known. I am not the Director. So I am not well-known

Coding: Variables

a: I am the Director

b: I am well-known

Coding the sentences:

1. $a \rightarrow b$
2. $\sim a$
3. $\sim b$

The final formula for deduction:

$((a \rightarrow b) \land \sim a) \rightarrow \sim b$

Proof or Otherwise

a	b	a → b	(a → b) Λ ~a	((a → b) Λ ~a) → ~b
T	T	T	F	T
T	F	F	F	T
F	T	T	T	F
F	F	T	T	T

Example: 3

If Rajat is the Director then Rajat is well-known. Rajat is the Director. So Rajat is well-known

Coding: Variables

a: Rajat is the Director

b: Rajat is well-known

Coding the sentences:

1. $a \rightarrow b$
2. a
3. b

The final formula for deduction:

$((a \rightarrow b) \wedge a) \rightarrow b$

Structure is identical to Example 1

Examples 4 and 5

- **If a cat is orange coloured then the cat is a foreign cat. Pussy-Cat is blue coloured. Therefore Pussy-Cat is not a foreign cat.**
- **Wherever Mary goes, so does the lamb. Mary goes to school. So the lamb goes to school.**

Beyond the scope of Propositional or Boolean Logic. We will take these up later

Examples 6 and 7

- If Asha is elected VP then Rajat is chosen as G-Sec and Bharati is chosen as Treasurer. Rajat is not chosen as G-Sec. Therefore Asha is not elected VP.
- If Asha is elected VP then Rajat is chosen as G-Sec and Bharati is chosen as Treasurer. Rajat is chosen as G-Sec. Therefore Asha is elected VP.

Examples 8 and 9

- If Asha is elected VP then Rajat is chosen as G-Sec or Bharati is chosen as Treasurer. Rajat is not chosen as G-Sec. Therefore if Asha is elected as VP then Bharati is chosen as Treasurer
- If Asha is elected VP then Rajat is chosen as G-Sec and Bharati is selected as Treasurer. Rajat is chosen as G-Sec. Therefore if Asha is elected as VP then Bharati is not chosen as Treasurer

Examples 10 and 11

- If Asha is elected VP then either Rajat is chosen as G-Sec or Bharati is chosen as Treasurer. Rajat is not chosen as G-Sec. Therefore if Asha is elected as VP then Bharati is chosen as Treasurer
- If Asha is elected VP then either Rajat is chosen as G-Sec and Bharati is selected as Treasurer. Rajat is chosen as G-Sec. Therefore if Asha is elected as VP then Bharati is not chosen as Treasurer

Example 12

- If the unicorn is mythical, then it is immortal, but if it is not mythical, then it is a mortal mammal.
- If the unicorn is either immortal or a mammal, then it is horned.
- The unicorn is magical if it is horned

- Can we prove that the unicorn is mythical? Magical? Horned?

Formula $((a \rightarrow b) \wedge a) \rightarrow b)$ is valid

a	b	$a \rightarrow b$	$(a \rightarrow b) \wedge a$	$((a \rightarrow b) \wedge a) \rightarrow b$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

Formula $((a \rightarrow b) \land \sim a) \rightarrow \sim b)$ is not valid but is satisfiable

a	b	$a \rightarrow b$	$(a \rightarrow b) \land \sim a$	$((a \rightarrow b) \land \sim a) \rightarrow \sim b$
T	T	T	F	T
T	F	F	F	T
F	T	T	T	F
F	F	T	T	T

Interpretation, Proof, Algorithms, Data Structures

- Valid, non-valid, Satisfiable, Unsatisfiable
- Decidable but NP-Hard
- Faster Methods for validity checking:- Tree Method
- Data Structures: Binary Decision Diagrams
- Modern SAT Solvers
- Symbolic Method: Natural Deduction
- Soundness and Completeness of a Method

Rules of Natural Deduction

- Modus Ponens: $(a \rightarrow b)$, a :- therefore b
- Modus Tollens: $(a \rightarrow b)$, ~b :- therefore ~a
- Hypothetical Syllogism: $(a \rightarrow b)$, $(b \rightarrow c)$:- therefore $(a \rightarrow c)$
- Disjunctive Syllogism: (a V b), ~a:- therefore b
- Constructive Dilemma: $(a \rightarrow b) \wedge (c \rightarrow d)$, (a V c) :- therefore (b V d)
- Destructive Dilemma: $(a \rightarrow b) \wedge (c \rightarrow d)$, (~b V ~d) :- therefore (~a V ~c)
- Simplification: $a \wedge b$:- therefore a
- Conjunction: a, b:- therefore $a \wedge b$
- Addition: a :- therefore a V b

Natural Deduction is Sound and Complete

THANK YOU