Constraint Satisfaction Problems: I
Introduction to CSP

Partha P Chakrabarti

Indian Institute of Technology Kharagpur

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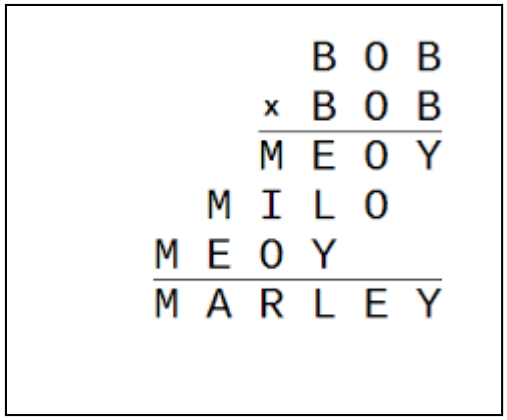
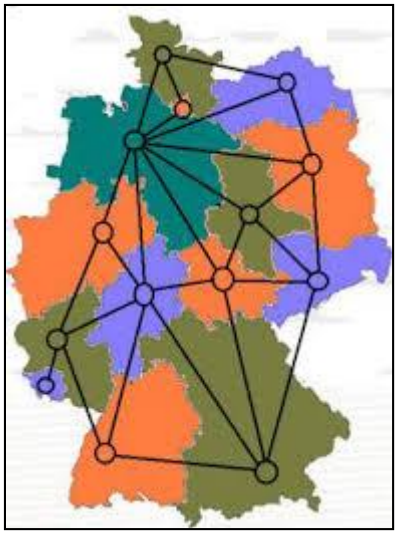
Sample Constraint Satisfaction Problems (CSP)

- Crossword Puzzle
- N-Queens on a Chess-Board
- Time-Table Preparation
- Flight Scheduling (Crew, Gate, Runway, etc)
- Object / Scene Labelling
- Cargo Packing
- Map Colouring
- Cryptic Puzzles
- Scheduling the Hubble Telescope
- Boolean Satisfiability (SAT)





Basic CSP Formulation

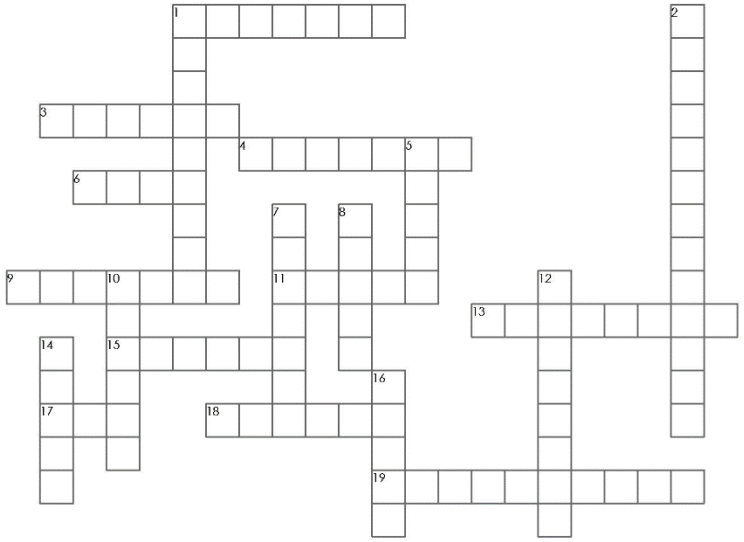
- **Variables**
  - A Finite Set of Variables V_1, V_2, …., V_n
- **Domains**
  - Each Variable has a Domain D_1, D_2, …., D_n from which it can take a value.
  - The Domains may be discrete or continuous domains
- **Satisfaction Constraints**
  - A Finite Set of Satisfaction Constraints, C_1, C_2, …C_m
  - Constraints may be unary, binary or be among many variables of the domain
  - All Constraints have a Yes / No Answer for Satisfaction given values of variables
- **Optimization Criteria (Optional)**
  - A Set of Optimization Functions O_1, O_2, ….O_p
  - These Optimization Functions are typically max or min type
- **Solution**
  - To Find a Consistent Assignment of Domain Values to each Variable so that All Constraints are Satisfied and the Optimization Criteria (if any) are met.

Example 1: Crossword Puzzle

Name: _____

Christmas Crosspatch Puzzle

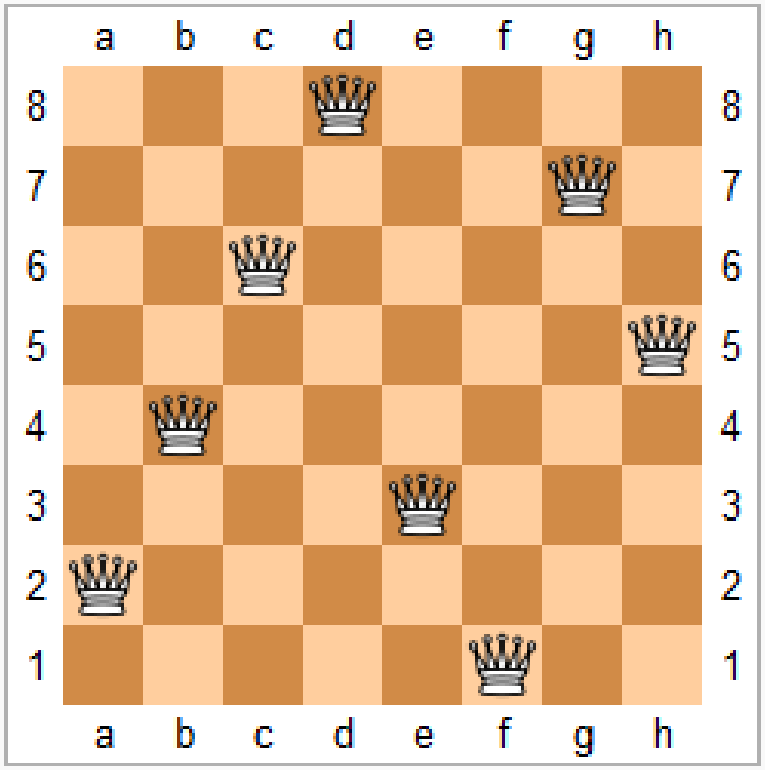
Crosspatch puzzle for brain challenge. Place the words in the word bank into the blank boxes in the grid below. Start with the word that has the most/least number of letters.



dolls	tidings	miracle	Blitzen
turkey	Dasher	happy	holly
reindeer	pie	wreath	family reunion
poinsettia	dressing	trips	yule
candles	ribbon	elves	mistletoe

1. VARIABLES
2. DOMAINS
3. SATISFACTION CONSTRAINTS
4. SOLUTION

Example 2: N-Queens on a Chess Board



One solution to the eight queens puzzle

1. VARIABLES
2. DOMAINS
3. SATISFACTION CONSTRAINTS
4. SOLUTION

Example 3: Airport Flight Gate Scheduling

International Departures

Flight No	Destination	Time	Gate	Remarks
CX7183	Berlin	7:50	A-11	Gate closing
QF3474	London	7:50	A-12	Gate closing
BA372	Paris	7:55	B-10	Boarding
AY6554	New York	8:00	C-33	Boarding
KL3160	San Francisco	8:00	F-15	Boarding
BA8903	Manchester	8:05	B-12	Gate lounge open
BA710	Los Angeles	8:10	C-12	Check-in open
QF3371	Hong Kong	8:15	F-10	Check-in open
MA4866	Barcelona	8:15	F-12	Check-in at kiosks
CX7221	Copenhagen	8:20	G-32	Check-in at kiosks

1. VARIABLES
2. DOMAINS
3. SATISFACTION CONSTRAINTS
4. OPTIMIZATION CRITERIA
5. SOLUTION

Example 4: Time Table Preparation

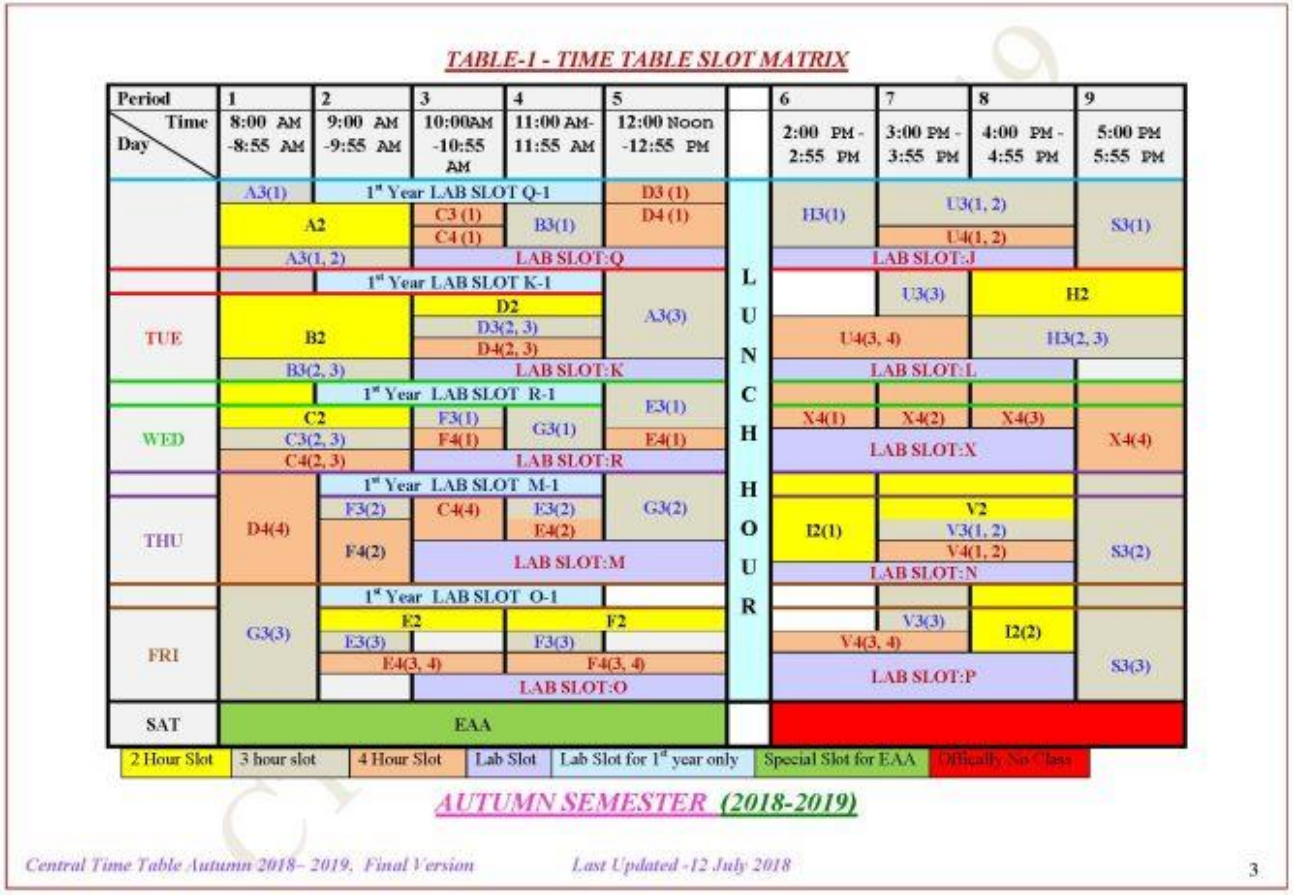


TABLE-1 - TIME TABLE SLOT MATRIX

Period / Time / Day	1 8:00 AM -8:55 AM	2 9:00 AM -9:55 AM	3 10:00AM -10:55 AM	4 11:00 AM-11:55 AM	5 12:00 Noon -12:55 PM		6 2:00 PM - 2:55 PM	7 3:00 PM - 3:55 PM	8 4:00 PM - 4:55 PM	9 5:00 PM 5:55 PM
	A3(1)	1st Year LAB SLOT Q-1			D3 (1)	LUNCH HOUR	H3(1)	U3(1, 2)		S3(1)
	A2		C3 (1)	B3(1)	D4 (1)			U4(1, 2)		
	A3(1, 2)		LAB SLOT:Q				LAB SLOT:J			
		1st Year LAB SLOT K-1			A3(3)			U3(3)	H2	
TUE	B2		D2				U4(3, 4)		H3(2, 3)	
	B3(2, 3)		D3(2, 3) D4(2, 3)				LAB SLOT:L			
		1st Year LAB SLOT R-1			E3(1)					
WED	C2		F3(1)	G3(1)			X4(1)	X4(2)	X4(3)	X4(4)
	C3(2, 3)		F4(1)		E4(1)		LAB SLOT:X			
	C4(2, 3)		LAB SLOT:R							
		1st Year LAB SLOT M-1			G3(2)					
THU	D4(4)	F3(2)	C4(4)	E3(2)			I2(1)	V2		S3(2)
		F4(2)		E4(2)				V3(1, 2)		
		LAB SLOT:M						V4(1, 2)		
							LAB SLOT:N			
	G3(3)	1st Year LAB SLOT O-1								
		E2		F2				V3(3)	I2(2)	
FRI		E3(3)		F3(3)			V4(3, 4)			S3(3)
		E4(3, 4)		F4(3, 4)						
		LAB SLOT:O					LAB SLOT:P			
SAT	EAA									

2 Hour Slot | 3 hour slot | 4 Hour Slot | Lab Slot | Lab Slot for 1st year only | Special Slot for EAA | Officially No Class

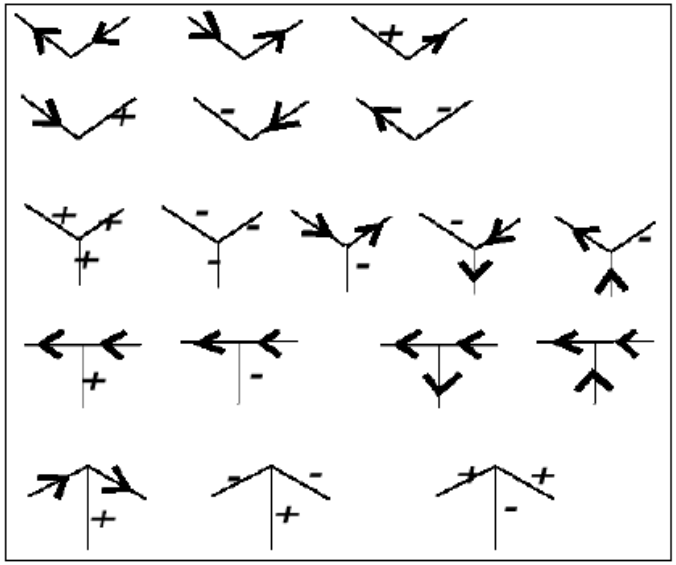
AUTUMN SEMESTER (2018-2019)

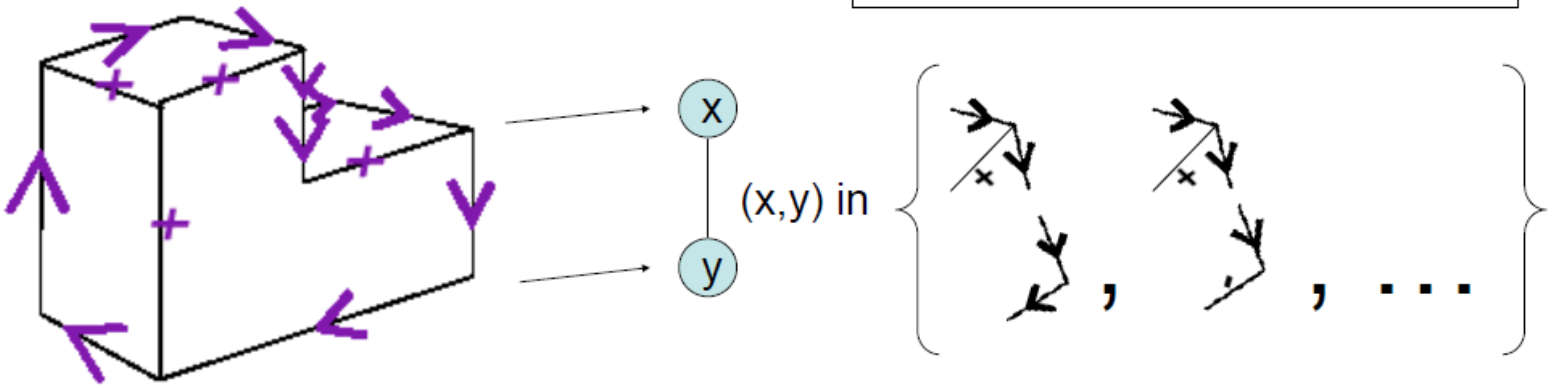
Central Time Table Autumn 2018- 2019, Final Version *Last Updated -12 July 2018* 3

1. VARIABLES
2. DOMAINS
3. SATISFACTION CONSTRAINTS
4. OPTIMIZATION CRITERIA
5. SOLUTION

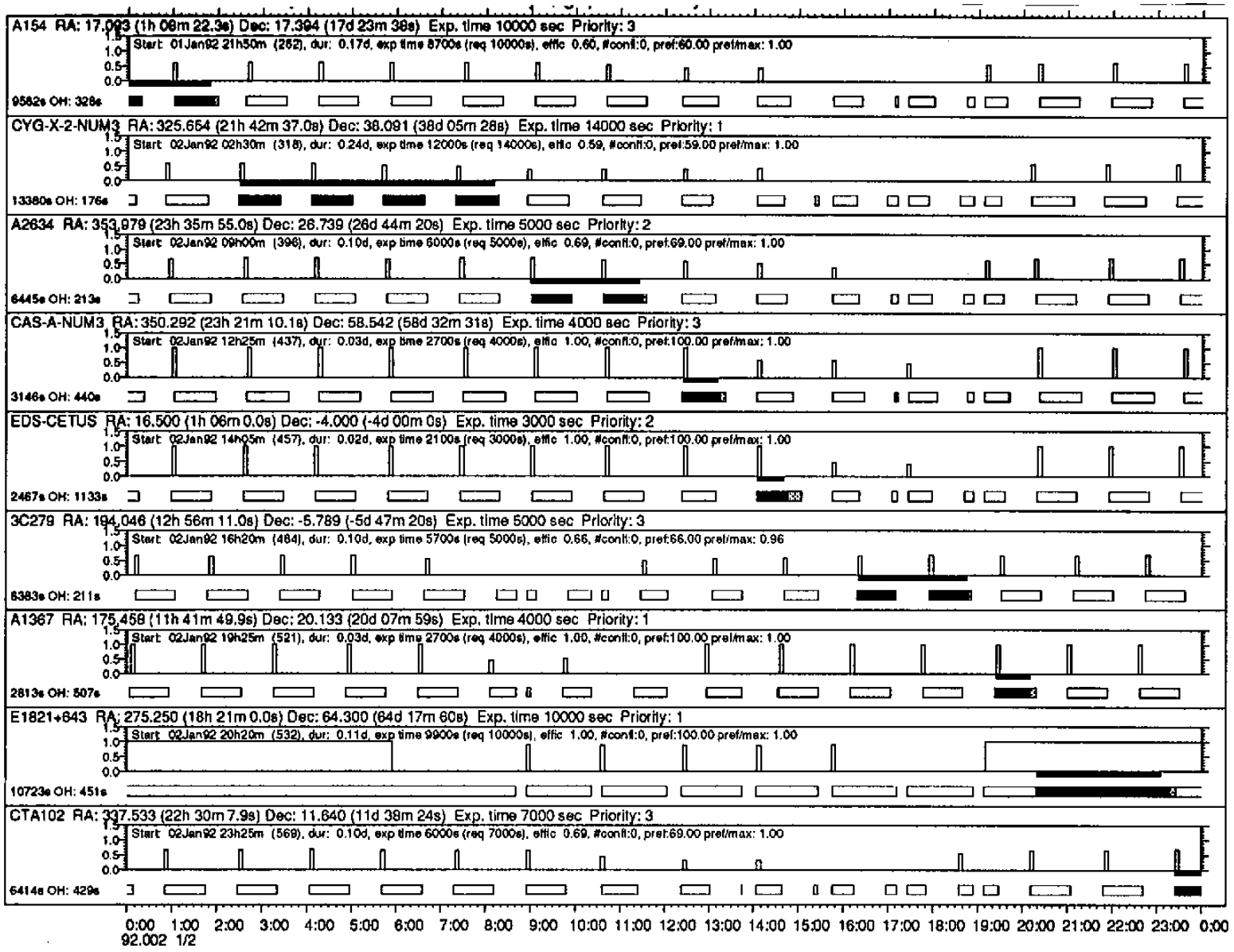
Example 5: Scene Labelling

- Only certain junctions are physically possible
- How can we formulate a CSP to label an image?
- Variables: vertices
- Domains: junction labels
- Constraints: both ends of a line should have the same label





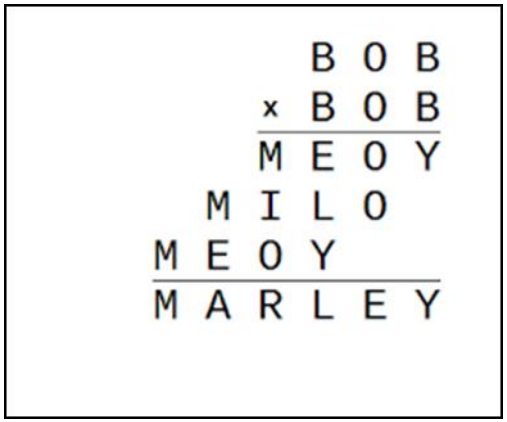
Example 6: Hubble Telescope



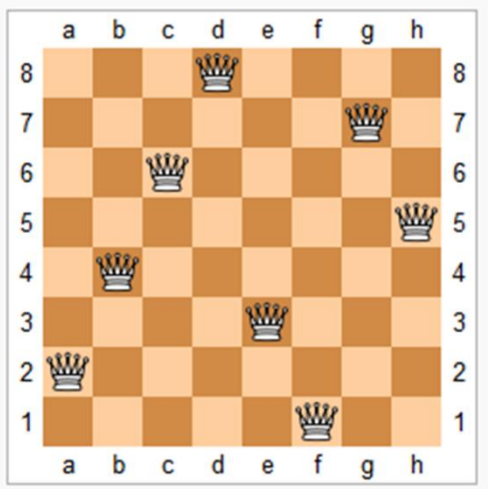
Solution Overview

- The following Basic Steps are followed:
  - CSP Graph Creation:
    - Create a Node for Every Variable. All possible Domain Values are initially Assigned to the Variable
    - Draw edges between Nodes if there is a Binary Constraint. Otherwise Draw a hyper-edge between nodes with constraints involving more than two variables
  - Constraint Propagation:
    - Reduce the Valid Domains of Each Variable by Applying Node Consistency, Arc / Edge Consistency, K-Consistency, till no further reduction is possible. If a solution is found or problem found to have no consistent solution, then terminate
  - Search for Solution:
    - Apply Search Algorithms to Find Solutions
    - There are interesting properties of CSP graphs which lead of efficient algorithms in some cases
    - Issues for Search: Backtracking Scheme, Ordering of Children, Forward Checking (Look-Ahead), Dynamic Constraint Propagation
    - Solving by Converting to SAT problems

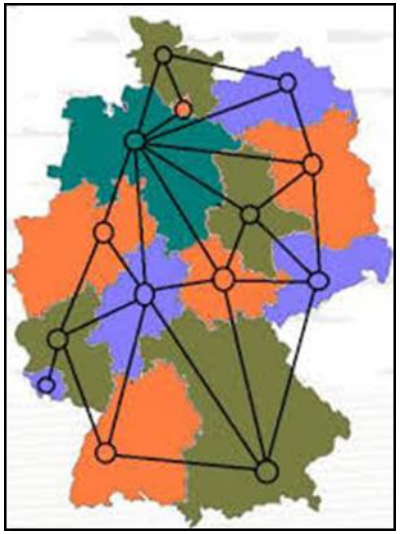
Modelling Using CSP



CRYPTARITHMETIC PUZZLE



N-QUEENS



MAP COLOURING

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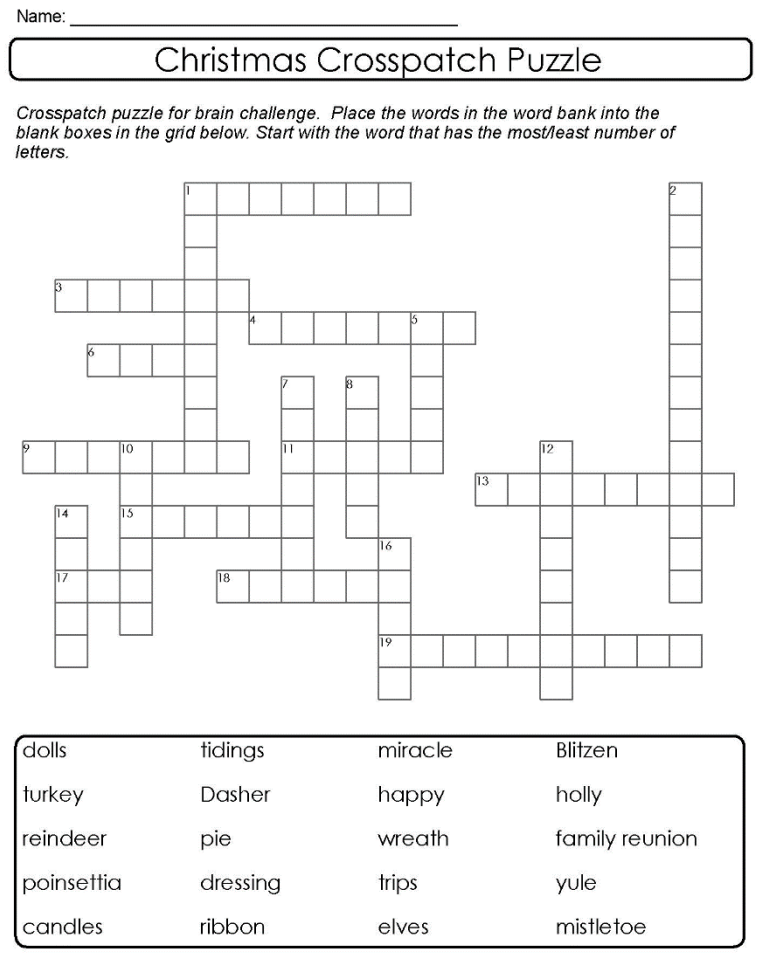
BA8903	Manchester	8:05	B-12	Gate lounge open
BA710	Los Angeles	8:10	C-12	Check-in open
QF3371	Hong Kong	8:15	F-10	Check-in open
MA4866	Barcelona	8:15	F-12	Check-in at kiosks
CX7221	Copenhagen	8:20	G-32	Check-in at kiosks

AIRPORT FLIGHT GATE SCHEDULING

Constraint Propagation Schemes

- **Constraints**
  - Unary Constraints or Node Constraints
  - Binary Constraints or Edges between CSP Nodes
  - Higher order or Hyper-Edges between CSP Nodes
- **Node Consistency**
  - For every Variable V_i, remove all elements of D_i that do not satisfy the Unary Constraints for the Variable
  - First Step is to reduce the domains using Node Consistency
- **Arc Consistency**
  - For every element x_ij of D_i, for every edge from V_i to V_j, remove x_ij if it has no consistent value(s) in other domains satisfying the Constraints
  - Continue to iterate using Arc Consistency till no further reduction happens.
- **K-Consistency or Path Consistency**
  - For every element y_ij of D_i, choose a Path of length L with L variables, use a consistency checking method similar to above to reduce domains if possible
- **Solution**
  - If any Node becomes empty, no solution exists
  - If every Node has a single element then we have a solution
  - Otherwise Search is deployed with Backtracking, Search Ordering, Dependency Directed Backtracking and Dynamic Constraint Propagation till one or all solutions are found if any.

Solving Crossword Puzzle using CSP



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turkey	Dasher	happy	holly
reindeer	pie	wreath	family reunion
poinsettia	dressing	trips	yule
candles	ribbon	elves	mistletoe

1. VARIABLES
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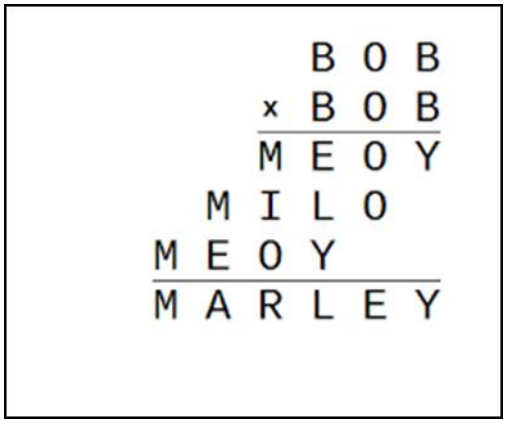
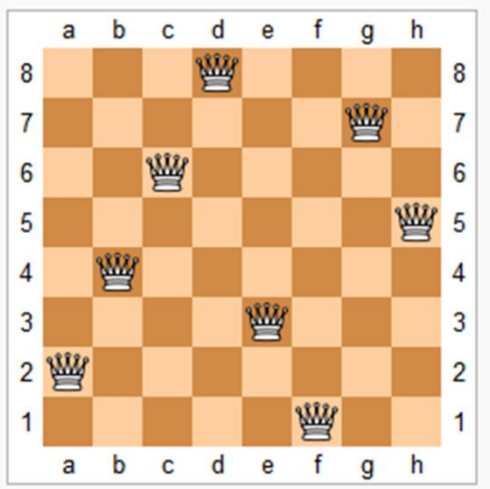
1. CSP Graph Formation
2. Constraint Propagation
   a) Node Consistency
   b) Arc Consistency
3. Solution Finding

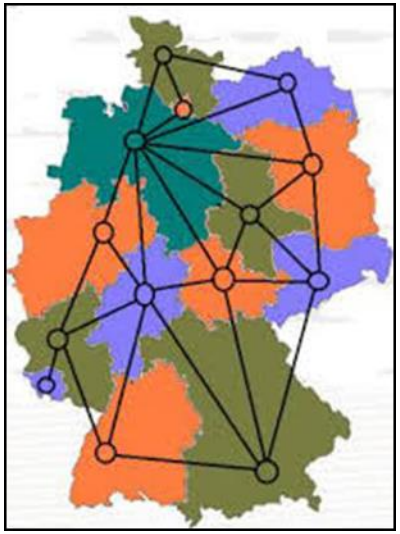
Solving More Examples Using CSP



CRYPTARITHMETIC PUZZLE



N-QUEENS



MAP COLOURING

AIRPORT FLIGHT GATE SCHEDULING

Thank you