Logical Deduction: II
Introduction to Predicate Logic

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Basic Examples

- **Wherever Mary goes, so does the lamb. Mary goes to school. So the lamb goes to school.**
- **No contractors are dependable. Some engineers are contractors. Therefore some engineers are not dependable.**
- **All dancers are graceful. Ayesha is a student. Ayesha is a dancer. Therefore some student is graceful.**
- **Every passenger is either in first class or second class. Each passenger is in second class if and only if he or she is not wealthy. Some passengers are wealthy. Not all passengers are wealthy. Therefore some passengers are in second class.**

Variables, Constants, Predicate Symbols and Two New Connectors

- Wherever Mary goes, so does the Lamb. Mary goes to School. So the Lamb goes to School.

Predicate: goes(x,y) to represent x goes to y

New Connectors: ∃ (there exists), ∀(for all)

F1: ∀x(goes(Mary, x) → goes(Lamb, x))

F2: goes(Mary, School)

G: goes(Lamb, School)

To prove: (F1 ∧ F2) → G) is always true

Example 2

No contractors are dependable. Some engineers are contractors. Therefore some engineers are not dependable.

Predicates: contractor(x), dependable(x), engineer(x)

F1: $\forall x(\text{contractor}(x) \rightarrow \sim\text{dependable}(x))$

[Alternative: $\sim\exists x\ (\text{contractor}(x) \wedge \text{dependable}(x))$]

F2: $\exists x(\text{engineer}(x) \wedge \text{contractor}(x))$

G: $\exists x(\text{engineer}(x) \wedge \sim\text{dependable}(x))$

To prove: $(F1 \wedge F2) \rightarrow G)$ is always true

Examples 3 and 4

- All dancers are graceful. Ayesha is a student. Ayesha is a dancer. Therefore some student is graceful.
- Every passenger is either in first class or second class. Each passenger is in second class if and only if the passenger is not wealthy. Some passengers are wealthy. Not all passengers are wealthy. Therefore some passengers are in second class.

More Examples with Quantifiers

- Someone likes everyone
- Everyone likes someone
- There is someone whom everyone likes
- Everyone likes everyone
- If everyone likes everyone else then someone likes everyone else
- If there is a person whom everyone likes then that person likes himself

Understanding the Scope Rules

An example with Function Symbols

- If x is greater than y and y is greater than z then x is greater than z.
- The age of a person is greater than the age of his child.
- Therefore the age of a person is greater than the age of his grandchild.
- Also the sum of ages of two children are never more than the sum of ages of their parents.

Variables and Symbols

- Variables, Free variables, Bound variables
- Symbols – proposition symbols, constant symbols, function symbols, predicate symbols
- Variables can be quantified in first order predicate logic
- Symbols cannot be quantified in first order predicate logic
- Interpretations are mappings of symbols to relevant aspects of a domain

Terminology for Predicate Calculus

- Domain: D
- Constant Symbols: M, N, O, P, ….
- Variable Symbols: x,y,z,….
- Function Symbols: F(x), G(x,y), H(x,y,z)
- Predicate Symbols: p(x), q(x,y), r(x,y,z),
- Connectors: ~, Λ, V, →, ∃, ∀
- Terms:
- Well-formed Formula:
- Free and Bound Variables:
- Interpretation, Valid, Non-Valid, Satisfiable, Unsatisfiable

Validity, Satisfiability and Structure

F1: $\forall x(\text{goes}(\text{Mary}, x) \rightarrow \text{goes}(\text{Lamb}, x))$

F2: goes(Mary, School)

G: goes(Lamb, School)

To prove: $(F1 \land F2) \rightarrow G)$ is always true

Is the same as:

F1: $\forall x(g(M, x) \rightarrow g(L, x))$

F2: $g(M, S)$

G: $g(L, S)$

To prove: $(F1 \land F2) \rightarrow G)$ is always true

Interpretations, Validity, Satisfiability

What is an Interpretation? **Assign a domain set D, map constants, functions, predicates suitably.** **The formula will now have a truth value**

Example:

F1: ∀x(g(M, x) → g(L, x))

F2: g(M, S)

G: g(L, S)

Interpretation 1: D = {Akash, Baby, Home, Play, Ratan, Swim}, etc.,

Interpretation 2: D = Set of Integers, etc.,

How many interpretations can there be?

To prove Validity, means (F1 ∧ F2) → G) is true under all interpretations

To prove Satisfiability means (F1 ∧ F2) → G) is true under at least one interpretation

The Power of Expression is also a Limitation for Automation

- **Russell's Paradox** (The barber shaves all those who do not shave themselves. Does the barber shave himself?)
  - There is a single barber in town.
  - Those and only those who do not shave themselves are shaved by the barber.
  - Who shaves the barber?

- Checking Validity of First order logic is undecidable but partially decidable (semi-decidable) {Robinson's Method of Resolution Refutation}

- Higher order predicate logic - can quantify symbols in addition to quantifying variables.

$$\forall p((p(0) \wedge (\forall x(p(x) \rightarrow p(S(x))) \rightarrow \forall y(p(y))$$

Examples

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Thank you