Logical Deduction: I

Introduction to Propositional Logic

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History of Logic and Deduction

Role in Artificial Intelligence and Design of Intelligent Agents

First Few Examples

- If I am the Director then I am well-known. I am the Director. So I am well-known
- If I am the Director then I am well-known. I am not the Director. So I am not well-known.
- If Rajat is the Director then Rajat is well-known. Rajat is the Director. So Rajat is well known.
- If a cat is orange coloured then the cat is a foreign cat. Pussy-Cat is blue coloured. Therefore Pussy-Cat is not a foreign cat.
- Wherever Mary goes, so does the lamb. Mary goes to school. So the lamb goes to school.

Propositional / Boolean Logic

- Boolean variables a, b, c, d, … which can take values true or false.
- Boolean formulae developed using well defined connectors ~, Λ, V, →, etc, whose meaning (semantics) is given by their truth tables.
- Codification of sentences of the argument into Boolean Formulae.
- Developing the Deduction Process as obtaining truth of a combined formula expressing the complete argument.
- Determining the Truth or Validity of the formula and thereby proving or disproving the argument and Analyzing its truth under various interpretations.

Example: 1

If I am the Director then I am well-known. I am the Director. So I am well-known

Coding: Variables

a: I am the Director

b: I am well-known

Coding the sentences:

1. **a → b**
2. **a**
3. **b**

The final formula for deduction:

((a → b) Λ a) → b

Proof or Otherwise

a	b	$a \rightarrow b$	$(a \rightarrow b) \wedge a$	$((a \rightarrow b) \wedge a) \rightarrow b$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

Example: 2

If I am the Director then I am well-known. I am not the Director. So I am not well-known

Coding: Variables

a: I am the Director

b: I am well-known

Coding the sentences:

1. **$a \rightarrow b$**
2. **$\sim a$**
3. **$\sim b$**

The final formula for deduction:

$((a \rightarrow b) \wedge \sim a) \rightarrow \sim b$

Proof or Otherwise

a	b	a → b	(a → b) Λ ~a	((a → b) Λ ~a) → ~b
T	T	T	F	T
T	F	F	F	T
F	T	T	T	F
F	F	T	T	T

Example: 3

If Rajat is the Director then Rajat is well-known.
Rajat is the Director. So Rajat is well-known

Coding: Variables

a: Rajat is the Director

b: Rajat is well-known

Coding the sentences:

1. $a \rightarrow b$
2. a
3. b

The final formula for deduction:

$((a \rightarrow b) \wedge a) \rightarrow b$

Structure is identical to Example 1

Examples 4 and 5

- If a cat is orange coloured then the cat is a foreign cat. Pussy-Cat is blue coloured. Therefore Pussy-Cat is not a foreign cat.
- Wherever Mary goes, so does the lamb. Mary goes to school. So the lamb goes to school.

Beyond the scope of Propositional or Boolean Logic. We will take these up later

Examples 6 and 7

- If Asha is elected VP then Rajat is chosen as G-Sec and Bharati is chosen as Treasurer. Rajat is not chosen as G-Sec. Therefore Asha is not elected VP.
- If Asha is elected VP then Rajat is chosen as G-Sec and Bharati is chosen as Treasurer. Rajat is chosen as G-Sec. Therefore Asha is elected VP.

Examples 8 and 9

- If Asha is elected VP then Rajat is chosen as G-Sec or Bharati is chosen as Treasurer. Rajat is not chosen as G-Sec. Therefore if Asha is elected as VP then Bharati is chosen as Treasurer
- If Asha is elected VP then Rajat is chosen as G-Sec and Bharati is selected as Treasurer. Rajat is chosen as G-Sec. Therefore if Asha is elected as VP then Bharati is not chosen as Treasurer

Examples 10 and 11

- If Asha is elected VP then either Rajat is chosen as G-Sec or Bharati is chosen as Treasurer. Rajat is not chosen as G-Sec. Therefore if Asha is elected as VP then Bharati is chosen as Treasurer
- If Asha is elected VP then either Rajat is chosen as G-Sec and Bharati is selected as Treasurer. Rajat is chosen as G-Sec. Therefore if Asha is elected as VP then Bharati is not chosen as Treasurer

Example 12

- **If the unicorn is mythical, then it is immortal, but if it is not mythical, then it is a mortal mammal.**
- **If the unicorn is either immortal or a mammal, then it is horned.**
- **The unicorn is magical if it is horned**

- **Can we prove that the unicorn is mythical? Magical? Horned?**

Formula (((a → b) Λ a) → b) is valid

a	b	a → b	(a → b) Λ a	((a → b) Λ a) → b
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

Formula (((a → b) Λ ~a) → ~b) is not valid but is satisfiable

a	b	a → b	(a → b) Λ ~a	((a → b) Λ ~a) → ~b
T	T	T	F	T
T	F	F	F	T
F	T	T	T	F
F	F	T	T	T

Interpretation, Proof, Algorithms, Data Structures

- Valid, non-valid, Satisfiable, Unsatisfiable
- Decidable but NP-Hard
- Faster Methods for validity checking:- Tree Method
- Data Structures: Binary Decision Diagrams
- Symbolic Method: Natural Deduction
- Soundness and Completeness of a Method

Rules of Natural Deduction

- Modus Ponens: (a → b), a :- therefore b
- Modus Tollens: (a → b), ~b :- therefore ~a
- Hypothetical Syllogism: (a → b), (b → c):- therefore (a → c)
- Disjunctive Syllogism: (a V b), ~a:- therefore b
- Constructive Dilemma: (a → b) Λ (c → d), (a V c) :- therefore (b V d)
- Destructive Dilemma: (a → b) Λ (c → d), (~b V ~d) :- therefore (~a V ~c)
- Simplification: a Λ b:- therefore a
- Conjunction: a, b:- therefore a Λ b
- Addition: a :- therefore a V b

Natural Deduction is Sound and Complete

THANK YOU