Knowledge Based Systems: Logic and Deduction

Course: CS40002
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Knowledge and Reasoning

- Representation, Reasoning and Logic
- Propositional Logic
- First-Order Logic
- Inference in first-order logic
  - Generalized Modus Ponens
  - Forward and backward chaining
  - Resolution
- Logical Reasoning Systems

The Wumpus World Environment

Adjacent means left, right, top, or bottom

- Stench: In squares containing and adjacent to wumpus
- Breeze: In squares adjacent to a pit

There can be one wumpus, one gold, and many pits. Agent starts from the bottom-left square of a grid.

The Wumpus World Environment

- The agent dies if it enters a square containing a pit or the wumpus
- The agent can shoot the wumpus along a straight line
- The agent has only one arrow

Logic

- A formal system for describing states of affairs, consisting of:
  - **Syntax: describes how to make sentences, and**
  - **Semantics: describes the relation between the sentences and the states of affairs**
- A proof theory – a set of rules for deducing the entailments of a set of sentences
- Improper definition of logic, or an incorrect proof theory can result in absurd reasoning

Types of Logics

Language	*What exists*	*Belief of agent*
Propositional Logic	Facts	T / F / Unknown
First-Order Logic	Facts, Objects, Relations	T / F / Unknown
Temporal Logic	Facts, Objects, Relations, Times	T / F / Unknown
Probability Theory	Facts	Degree of belief [0..1]
Fuzzy Logic	Degree of truth	Degree of belief [0..1]

Propositional Logic

- Given a set of atomic propositions $\mathcal{AP}$

Sentence $\rightarrow$ Atom | ComplexSentence

Atom $\rightarrow$ True | False | $\mathcal{AP}$

ComplexSentence $\rightarrow$ (Sentence)
| Sentence Connective Sentence
| $\neg$ Sentence

Connective $\rightarrow$ $\wedge$ | $\vee$ | $\Leftrightarrow$ | $\Rightarrow$

Inference Rules

- Modus Ponens or Implication Elimination:

$$\frac{\alpha \Rightarrow \beta, \quad \alpha}{\beta}$$

- Unit Resolution:

$$\frac{\alpha \vee \beta, \quad \neg\beta}{\alpha}$$

Inference Rules

- Resolution:

$$\frac{\alpha \vee \beta, \quad \neg\beta \vee \gamma}{\alpha \vee \gamma}$$

or

$$\frac{\neg\alpha \Rightarrow \beta, \quad \beta \Rightarrow \gamma}{\neg\alpha \Rightarrow \gamma}$$

…. and several other rules

Automated Reasoning

- If the unicorn is mythical, then it is immortal, but if it is not mythical, then it is a mortal mammal.
- If the unicorn is either immortal or a mammal, then it is horned.
- The unicorn is magical if it is horned

Can we prove that the unicorn is mythical? Magical? Horned?

Modeling in propositional logic

Propositions:

- Umyth: Unicorn in mythical
- Umort: Unicorn is mortal
- Umam: Unicorn is mammal
- Umag: Unicorn is magical
- Uhorn: Unicorn is horned

Automated Reasoning

- In general, the inference problem is NP-complete (Cook's Theorem)

- If we restrict ourselves to Horn sentences, then repeated use of Modus Ponens gives us a polytime procedure. Horn sentences are of the form:

$$P_1 \wedge P_2 \wedge \ldots \wedge P_n \Rightarrow Q$$

First-order Logic

- Constant →
  A | 5 | Kolkata | …
- Variable →
  a | x | s | …
- Predicate →
  Before | HasColor | Raining | …
- Function →
  Mother | Cosine | Headoflist | …

First-order Logic

- Sentence → AtomicSentence
  | Sentence Connective Sentence
  | Quantifier Variable, … Sentence
  | ¬ Sentence | (Sentence)
- AtomicSentence →
  Predicate(Term, …) | Term = Term
- Term →
  Function(Term, …) | Constant | Variable
- Connective → ⇒ | ∧ | ∨ | ⇔
- Quantifier → ∀ | ∃

Examples

- Not all students take both History & Biology
- Only one student failed History
- Only one student failed both History & Biology
- The best score in History is better than the best score in Biology
- No person likes a professor unless the professor is smart
- Politicians can fool some of the people all the time, and they can fool all the people some of the time, but they cant fool all the people all the time

Examples

- Russel's Paradox:
  - There is a single barber in town.
  - Those and only those who do not shave themselves are shaved by the barber.
  - Who shaves the barber?