CS 60030: Formal Systems
Spring Semester: 2018-2019
Tutorial 2
Linear Time Properties and ω-Regular Languages

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1. Consider the Transition System outlined below.

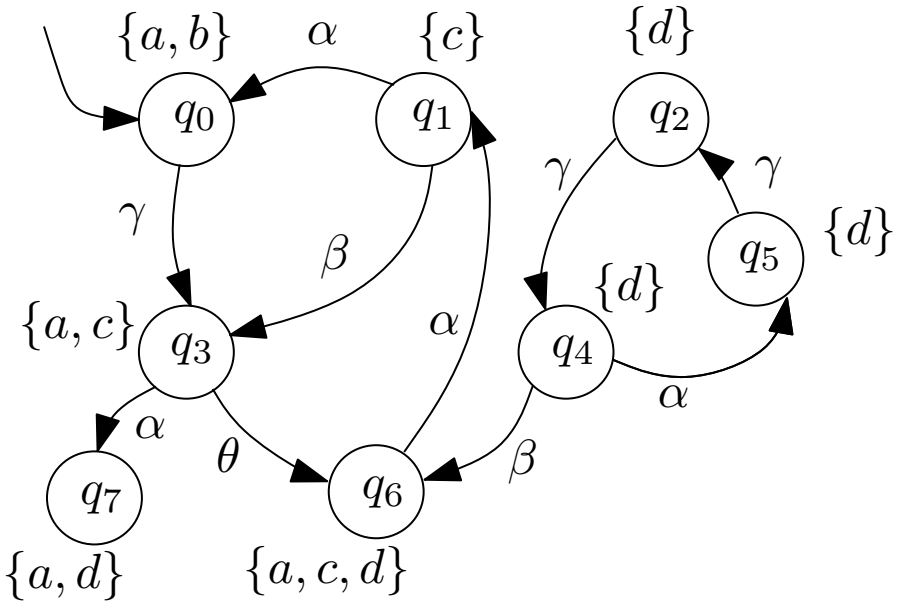

Figure: Transition System: TS_1

With respect to TS_1 answer the following questions:

1.1 List two finite non-initial, initial non-maximal execution fragments.

1.2 List any maximal execution fragment.

2. For each property φ_i, does $TS_1 \models \varphi_i$? If not, produce a counterexample trace.
    - 2.1 $\varphi_1 = \forall_{i \geq 0}\ a \in A_i$
    - 2.2 $\varphi_2 = \exists_{i \geq 0}\ a \notin A_i$
    - 2.3 $\varphi_3 = \exists_{i \geq 0}\ b \in A_i$
    - 2.4 $\varphi_4 = \exists_{i \geq 0} \forall_{j \geq i}\ c \in A_j$
    - 2.5 $\varphi_5 = \forall_{i \geq 0}\ a \vee c \in A_i$
    - 2.6 $\varphi_6 = \exists_{i \geq 0} \forall_{j \geq i}\ d \in A_j$
3. Which of the above properties are invariants? Which are safety properties but not invariants? Which are neither?

4. Write the ω-Regular Expressions for the following statements:
   - 4.1 A message is sent successfuly infinitely often. $\Sigma = \{S, T, F\}$
   - 4.2 Every time the process tries to send a message, it eventually succeeds in sending it. $\Sigma = \{S, T, F\}$
   - 4.3 The LED eventually turns red $\Sigma = \{R, G, B\}$.
   - 4.4 Whenever the LED is Blue, it eventually turns green.
   - 4.5 Either the LED is always Red or it alternates between Green and Blue.
   - 4.6 The invariant, $a \vee \neg b$ over $AP = \{a, b, c\}$.
   - 4.7 Process P visits the critical section infinitely often. $AP = \{wait, crit\}$.
   - 4.8 Consider $AP = \{a, b\}$, where the language has words of the form $A_0 A_1 ... \in (2^{AP})^{\omega}$, and is defined as follows

     $\exists^{\infty} j \geq 0.(a \in A_j \wedge b \in A_j)$ and $\exists j \geq 0.(a \in A_j \wedge b \notin A_j)$

5. For each of the following pairs, determine if they are equivalent. If the two are found to be not equivalent provide a string to distinguish between the two languages.
    - 5.1 $(A^*+B^*).C^\omega$ and $A^*.C^\omega + B^*.C^\omega$
    - 5.2 $(A+B)^*(C+D)^\omega$ and $(A+B)^*.C^\omega + (A+B)^*.D^\omega$
    - 5.3 $(A^* + B.C)^+.(C.C^*)^\omega$ and $(A^* + B.C)^+.(C)^\omega$
    - 5.4 $(A^*B)^\omega$ and $A^*.B^\omega$