CS 60030: Formal Systems
Spring Semester: 2018-2019
Tutorial 2
Linear Time Properties and ω-Regular Languages

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1. Consider the Transition System outlined below.

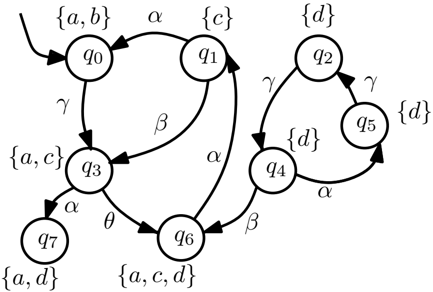

Figure: Transition System: TS_1

With respect to TS_1 answer the following questions:

1.1 List two finite non-initial, initial non-maximal execution fragments.

1.2 List any maximal execution fragment.

2. For each property φ_i, does $TS_1 \models \varphi_i$? If not, produce a counterexample trace.
   2.1 $\varphi_1 = \forall_{i \geq 0}\ a \in A_i$
   2.2 $\varphi_2 = \exists_{i \geq 0}\ a \notin A_i$
   2.3 $\varphi_3 = \exists_{i \geq 0}\ b \in A_i$
   2.4 $\varphi_4 = \exists_{i \geq 0} \forall_{j \geq i}\ c \in A_j$
   2.5 $\varphi_5 = \forall_{i \geq 0}\ a \vee c \in A_i$
   2.6 $\varphi_6 = \exists_{i \geq 0} \forall_{j \geq i}\ d \in A_j$
3. Which of the above properties are invariants? Which are safety properties but not invariants? Which are neither?

4. Write the ω-Regular Expressions for the following statements:
   - 4.1 A message is sent successfuly infinitely often. $\Sigma = \{S, T, F\}$
   - 4.2 Every time the process tries to send a message, it eventually succeeds in sending it. $\Sigma = \{S, T, F\}$
   - 4.3 The LED eventually turns red $\Sigma = \{R, G, B\}$.
   - 4.4 Whenever the LED is Blue, it eventually turns green.
   - 4.5 Either the LED is always Red or it alternates between Green and Blue.
   - 4.6 The invariant, $a \vee \neg b$ over $AP = \{a, b, c\}$.
   - 4.7 Process P visits the critical section infinitely often. $AP = \{wait, crit\}$.
   - 4.8 Consider $AP = \{a, b\}$, where the language has words of the form $A_0 A_1 ... \in (2^{AP})^{\omega}$, and is defined as follows

$$\exists^{\infty} j \geq 0.(a \in A_j \wedge b \in A_j) \text{ and } \exists j \geq 0.(a \in A_j \wedge b \notin A_j)$$

$$\overset{\infty}{\exists} \phi \equiv \forall_{k \geq 0} \exists_{j \geq k} \phi$$

4.1) $\left[(T+F)^{*}.\,S\right]^{\omega}$

4.2) $\left[T\cdot(F\cdot T)^{*}\cdot S \;+\; (F+S)\right]^{\omega}$

4.3) $(G+B)^{*}.\,R\cdot(R+G+B)^{\omega}$

4.4) $(B^{+}.\,G+R+G)^{\omega}$

4.5) $R^{\omega}+(G\cdot B)^{\omega}+(B\cdot G)^{\omega}$

4.6) For invariant $a \vee \neg b$ over $AP=\{a,b,c\}$

$\Sigma = 2^{AP} = \{\phi, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\}\}$

$= \{S_e, S_a, S_b, S_c, S_{ab}, S_{ac}, S_{bc}, S_{abc}\}$

$(S_e+S_a+S_c+S_{ab}+S_{ac}+S_{abc})^{\omega}$

4.7) $(wait^{*}.\,crit)^{\omega}$

4.8) $true^{*}(\{a\}+\{\ \})\cdot(true^{*}.\,\{a,b\})^{\omega}$

5. For each of the following pairs, determine if they are equivalent. If the two are found to be not equivalent provide a string to distinguish between the two languages.

   5.1 $(A^*+B^*).C^\omega$ and $A^*.C^\omega + B^*.C^\omega$
   5.2 $(A+B)^*(C+D)^\omega$ and $(A+B)^*.C^\omega + (A+B)^*.D^\omega$
   5.3 $(A^* + B.C)^+.(C.C^*)^\omega$ and $(A^* + B.C)^+.(C)^\omega$
   5.4 $(A^*B)^\omega$ and $A^*.B^\omega$

Let the first language of the pair be $\mathcal{L}_1$ and the second be $\mathcal{L}_2$.

5.1> Not Equivalent : $(A\cdot B)^+ C^\omega \in \mathcal{L}_1$; $(A\cdot B)^+ C^\omega \notin \mathcal{L}_2$

5.2> Not Equivalent : $(C\cdot D)^\omega \in \mathcal{L}_1$; $(C\cdot D)^\omega \notin \mathcal{L}_2$

5.3> Equivalent : $(C\cdot C^*)^\omega \equiv (C)^\omega$

5.4> Not Equivalent : $(AB)^\omega \in \mathcal{L}_1$; $(AB)^\omega \notin \mathcal{L}_2$