Tutorial 2

Foundations of Computing Science

Pallab Dasgupta
Professor,
Dept. of Computer Sc & Engg

Questions

1. Write Context Free Grammar for the following languages :
   a) {w|w contains at least three 1s} , $\sum$={0,1}
   b) {w|w starts and ends with the same symbol}
   c) {w| the length of w is odd}
   d) {w| the length of w is odd and its middle symbol is a 0}
   e) The set of strings over the alphabet {a,b} with more a's than b's
   f) L= $\{a^n b^{2n} c^k \mid n, k \geq 1\}$
   g) $\{w\#x \mid w^R$ is a substring of x for w, x $\in \{0,1\}^*\}$

2. Draw the PDA for the following languages:
   a) L1 = $\{ a^m w c w^r b^m \mid w \in \{0, 1\}+$ and $m \geq 1\}$
   b) L2 = $\{a^n b^m \mid m \leq n\}$
   c) The set of palindromes over {a, b}

3. For the language :

$A=\{a^i b^j c^k \mid i=j \text{ or } j=k \text{ where } i,j,k>=0\}$

a) Give a context-free grammar that generates A

b) Show that the language A is inherently ambiguous

4. Let CFG G be the following grammar.

S -> aSb | bY| Ya

Y -> bY|aY| ε

Give a simple description of L(G) in English. Use that description to give a CFG for the complement of L(G)

5. Convert the following CFG into an equivalent CFG in Chomsky Normal Form

A->BAB | B | ε

B-> 00 | ε

6. Using the pumping lemma for CFL, show that the following languages (A, B, C and D) are not context-free:

a) $A = \{ 0^n1^n0^n1^n \mid n \geq 0 \}$

b) $B = \{ 0^n\#0^{2n}\#0^{3n} \mid n \geq 0 \}$

c) $C = \{ w\#t \mid w \text{ is a substring of } t, \text{ where } w,t \in \{a,b\}^* \}$

d) $D = \{ wtw^R \mid w,t \in \{0,1\}^* \text{ and } |w| = |t| \}$

7. Prove the following:

(a) Let C be a context-free language and R be a regular Language. Prove that the language C $\cap$ R is context-free

(b) Let A={w|w $\in$ {a,b,c}* and w contains equal number of a's, b's and c's}. Use (a) to show that A is not a CFL.