Tutorial 1

Foundations of Computing Science

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Questions

1. Construct a DFA that will accept the following languages over the alphabet {0, 1}

(a) The set of all strings whose binary interpretation is divisible by three.

(b) All strings that start with 0 and has odd length or start with 1 and has even length.

(c) The set of all strings with an equal number of 0's and 1's, such that no prefix has two more 0's than 1's, nor two more 1's than 0's.

(d) The set of all strings with no of 0's divisible by 3 and no of 1's divisible by 2.

(e) All strings containing exactly three 0's and at least two 1's.

2. Write regular expressions for the following languages

(a) The set of all strings of 0’s and 1’s whose fourth symbol from the right end is 1.

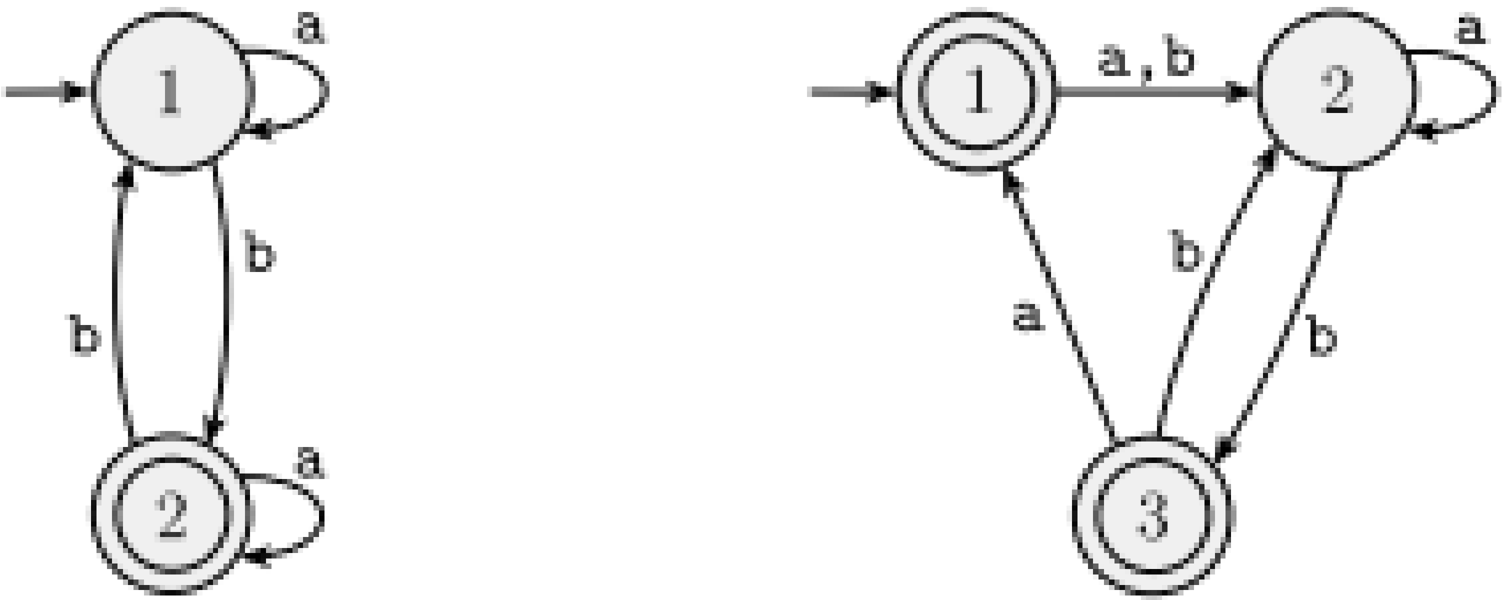
(b) The set of all strings with an equal number of 0’s and 1’s, such that no prefix has two more 0’s tan 1’s, nor two more 1’s than 0’s.

(c) The set of all strings of 0’s and 1’s whose number of 0s divisible by three.

(d) The set of all strings of 0’s and 1’s with at most one pair of consecutive 1’s.

(e) The set of all strings of 0’s and 1’s with at least one 0 and one 1.

3. Use the procedure described in Class to convert the following finite automata to regular expressions.



(a) (b)

4. In certain programming languages, comments appear between delimiters such as /# and #/. Let C be the language of all valid delimited comment strings. A member of C must begin with /# and end with #/ but have no intervening #/. For simplicity, assume that the alphabet for C is Σ = {a, b, /, #}. a. Give a DFA that recognizes C. b. Give a regular expression that generates C.

5. We define the avoids operation for languages A and B to be A avoids B = {w| w ∈ A and w doesn't contain any string in B as a substring}. Prove that the class of regular languages is closed under the avoids operation.

6. Let A/B = {w| wx ∈ A for some x ∈ B}. Show that if A is regular and B is any language, then A/B is regular.

7. Let x and y be strings and let L be any language. We say that x and y are distinguishable by L if some string z exists whereby exactly one of the strings xz and yz is a member of L; otherwise, for every string z, we have $xz \in L$ whenever $yz \in L$ and we say that x and y are indistinguishable by L. If x and y are indistinguishable by L, we write $x \equiv_L y$. Show that $\equiv_L$ is an equivalence relation.

8. Refer to Problem 7. Let L be a language and let X be a set of strings. Say that X is pairwise distinguishable by L if every two distinct strings in X are distinguishable by L. Define the index of L to be the maximum number of elements in any set that is pairwise distinguishable by L. The index of L may be finite or infinite.

a. Show that if L is recognized by a DFA with k states, L has index at most k.

b. Show that if the index of L is a finite number k, it is recognized by a DFA with k states.

c. Conclude that L is regular iff it has finite index. Moreover, its index is the size of the smallest DFA recognizing it.

9. One of the following languages is regular, and the other one is not regular. Identify both with respective proofs:

$$L_a = \{a^i b^j \mid i,j >= 0 \text{ and } i+j >= 10\}$$

$$L_b = \{a^i b^j \mid i,j >= 0 \text{ and } i-j >= 10\}$$

10. Let A be a non-empty language over alphabet {0,1}. Define

$$A^+ = \{\alpha 0 \beta \in \{0,1\}^* \mid \alpha\beta \in A\}$$

a. Give an example where $A^+ \subseteq A$, and $A^+ \not\subset A$

b. For any non empty language A, $A^+ \neq A$

11. Use the pumping lemma to prove these languages non-regular.

a. $L_1=\{1^*w$, w has fewer 1s than the first part$\}$

b. $L_2=\{ww^r$, where w^r is the reverse of w$\}$