Decidability

Foundations of Computing Science

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Decidable Problems Concerning Regular Languages

Theorems:

- Let A_{DFA} = {<B, w> | B is a DFA that accepts input string w},
  then A_{DFA} is a decidable language

- Let A_{NFA} = {<B, w> | B is a NFA that accepts input string w},
  then A_{NFA} is a decidable language

- Let A_{REX} = {<R, w> | R is a regular expression that generates string w},
  then A_{REX} is a decidable language

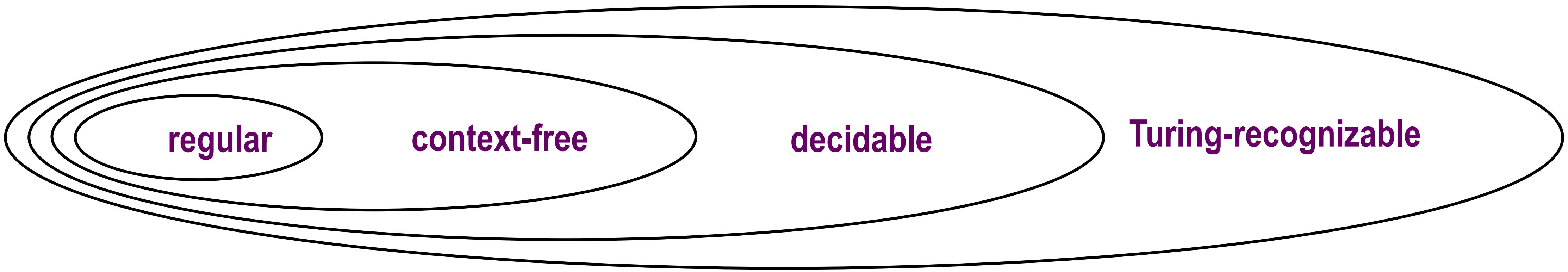
- Let E_{DFA} = {<A> | A is a DFA and L(A) = Φ},
  then E_{DFA} is a decidable language (*emptiness testing*)

- Let EQ_{DFA} = {<A, B> | A & B are DFAs and L(A) = L(B)},
  then EQ_{DFA} is a decidable language

Decidable Problems Concerning Context-free Languages

Theorems:

- Let $A_{CFG} = \{\langle G, w \rangle \mid G \text{ is a CFG that generates string } w\}$, then A_{CFG} is a decidable language
- Let $E_{CFG} = \{\langle G \rangle \mid G \text{ is a CFG and } L(G) = \Phi\}$, then E_{CFG} is a decidable language (*emptiness testing*)
- Let $EQ_{CFG} = \{\langle G, H \rangle \mid G \text{ \& } H \text{ are CFGs and } L(G) = L(H)\}$, then EQ_{CFG} is an undecidable language
- Every context-free language is decidable



The relationship among classes of languages

The Halting Problem

- Let $A_{TM} = \{\langle M, w\rangle \mid M \text{ is a TM and } M \text{ accepts } w\}$
  - A_{TM} is *Turing-recognizable*
  - A_{TM} is *undecidable*
- The Diagonalization Method [Georg Cantor, 1873]
  - Definitions:
    - A function that is both one-to-one and onto is called a *correspondence*
    - A set is *countable* if either it is finite or it has the same size as $\mathcal{N}$
  - Example (Theorem): The set of real numbers ($\mathcal{R}$) is *uncountable*
  - Corollary: Some languages are not *Turing-recognizable*
- The Halting Problem (A_{TM}) is *undecidable*
- A Turing-unrecognizable Language
  - Theorem: A language is *decidable* if and only if it is *Turing-recognizable* and *co-Turing-recognizable*
  - Corollary: $\overline{A_{TM}}$ is *not* Turing-recognizable