Foundation of Computer Science (CS60001)
Solution-06

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1. Which of the following problems are decidable and which are not decidable. Explain your answer.

   (a) Given a Turing machine M, a state q and a string w, whether M ever reaches state q when started with input w from its initial state.

   Solution : Undecidable because otherwise we can solve the acceptance problem of TM as follows: "Input ⟨M, w⟩ where M is a TM and w is the input string: For each accept state q_a of M, run the decider of the above problem on he input ⟨M, w, q_a ⟩; if the decider ever yields (yes) , stop with (yes) ; else, if it stops with (no) answer for all these runs, answer (no) ."

   (b) Given a TM M, whether M ever writes a non blank symbol when started on the empty tape.

   Solution: Let the machine only writes blank symbol. Then its number of configurations in the com computation on w is q × 2, where q is the number of states of M; the factor 2 is for the choices re. the direction of heads movement; there is no factor for the written symbol because that is always blank. So the problem is decidable, decided by the following machine: input ⟨M,w⟩, run M on w for q × 2 steps; if it M ever writes a non blank symbol, stop with yes answer; if M never writes a non blank symbol, stop with no answer.

   (c) Given a TM M and a string w, whether M moves its head to the left when started with input w.

   Solution : Let q be the number of states of the input TM M. Let $\mid \Sigma \cup \Gamma \mid$ be the cardinality of the union of the input and the auxiliary alphabet. If M never moves to the left, then it has only q × ($\mid \Sigma \cup \Gamma \mid +1$) configurations because at each step, the choice is to write one of the symbols of $\mid \Sigma \cup \Gamma \mid$ or the blank symbol. There is no choice re. the direction of the move. Also, if w = σ_1 $\sigma_2 ... \sigma_k$, then the tape pattern keeps decreasing in length at each step because the left side of the head is not of concern. So the problem is decidable decided by the following machine: "input ⟨ M, w ⟩, run M on w for q × $\mid \Sigma \cup \Gamma \mid$ steps; if it M ever moves to the right, stop with (yes) answer; if M never takes a left move, stop with (no) answer."

2. Prove that finite machine with 2 push down store is same powerful as turing machine

   Solution: Zohar Manna 1st Chapter

3. Show that the Post Correspondence Problem is decidable over the unary alphabet $\Sigma = \{1\}$.

   Solution: Let P = $\{\langle\ \alpha_i,\ \beta_i\ \rangle$, $1 \leq$ i $\leq$ k $\}$ be a Post System over $\Sigma = \{1\}$. In this case the members of the pairs can be looked upon as nonnegative numbers in unary encoding. The concatenation of two strings α_i and α_j is only addition of the respective numbers. Hence, if there is a solution, then any of its permutations is also a solution. Secondly, the pairs which have members having identical length are not important because even if one such pair is there, we have a solution comprising a single integer corresponding to that pair. When there is no equal pair, what is important is the difference between the lengths (i.e., the magnitudes, in unary encoding) of the members in a pair; for two distinct pairs, $\langle\ \alpha_i,\ \beta_i\ \rangle$ and $\langle\ \alpha_j,\ \beta_j\ \rangle$, let$\alpha_i \ominus \beta_i$ = m, m > 0 and $\alpha_j \ominus \beta_j$ = n, n > 0; then it is possible to find a solution (an integer sequence) by finding the *LCM(m, n)*. Let l = *LCM(m, n)/m* and p = *LCM(m, n)/n*. Obviously, *l.m = p.n* which means that the integer sequence $\langle$ i^{l}, $\mathrm{j}^{\mathrm{p}}\rangle$ (i.e i repeated l times followed by j repeated p times) will be a solution. Since any combination of negative and positive numbers will result in some LCM, the solution exists as long as both classes of pairs exist, one having shorter first members and the other having larger members. So the algorithm checks if there is a pair having equal length strings; if so it accepts. Else, it ensures that if there is a pair belonging to one class, there should be a pair belonging to the second class. If so, it accepts the input Post System, else if there are some pairs in one class but no pair in the other class, then it rejects.