Decidability

CS60001: Foundations of Computing Science

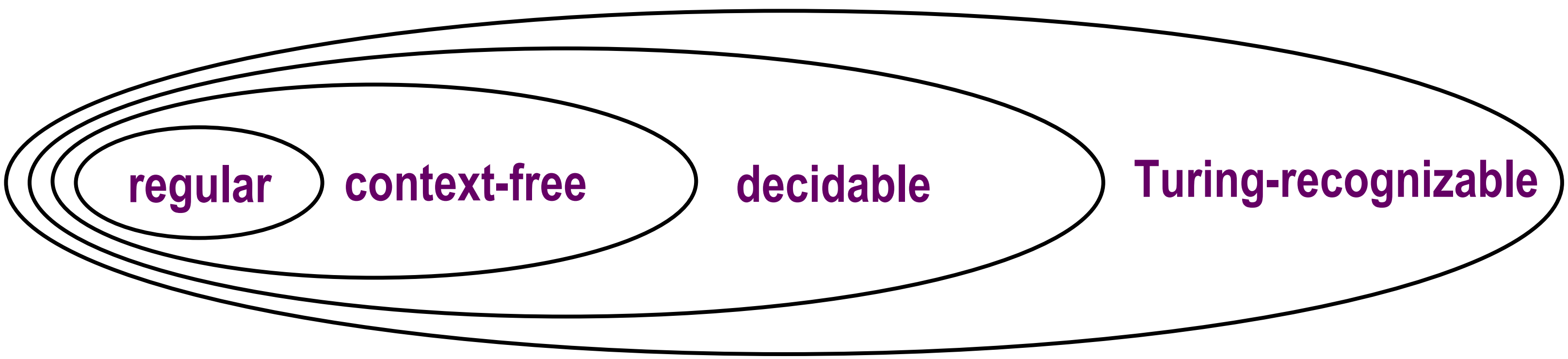
Pallab Dasgupta
Professor, Dept. of Computer Sc. & Engg.,
Indian Institute of Technology Kharagpur

Decidable Problems Concerning Regular Languages

- **Theorems:**
  - Let A_{DFA} = {<B, w> | B is a DFA that accepts input string w}, then A_{DFA} is a decidable language
  - Let A_{NFA} = {<B, w> | B is a NFA that accepts input string w}, then A_{NFA} is a decidable language
  - Let A_{REX} = {<R, w> | R is a regular expression that generates string w}, then A_{REX} is a decidable language
  - Let E_{DFA} = {<A> | A is a DFA and L(A) = Φ}, then E_{DFA} is a decidable language (*emptiness testing*)
  - Let EQ_{DFA} = {<A, B> | A & B are DFAs and L(A) = L(B)}, then EQ_{DFA} is a decidable language

Decidable Problems Concerning Context-free Languages

- **Theorems:**
  - Let A_{CFG} = {<G, w> | G is a CFG that generates string w}, then A_{CFG} is a decidable language
  - Let E_{CFG} = {<G> | G is a CFG and L(G) = Φ}, then E_{CFG} is a decidable language (*emptiness testing*)
  - Let EQ_{CFG} = {<G, H> | G & H are CFGs and L(G) = L(H)}, then EQ_{CFG} is a decidable language
  - Every context-free language is decidable



The relationship among classes of languages

The Halting Problem

- Let $A_{TM} = \{\langle M, w\rangle \mid M \text{ is a TM and } M \text{ accepts } w\}$
  - A_{TM} is *Turing-recognizable*
  - A_{TM} is *undecidable*
- The Diagonalization Method [Georg Cantor, 1873]
  - Definitions:
    - A function that is both one-to-one and onto is called a *correspondence*
    - A set is *countable* if either it is finite or it has the same size as $\mathcal{N}$
  - Example (Theorem): The set of real numbers ($\mathcal{R}$) is *uncountable*
  - Corollary: Some languages are not *Turing-recognizable*
- The Halting Problem (A_{TM}) is *undecidable*
- A Turing-unrecognizable Language
  - Theorem: A language is *decidable* if and only if it is *Turing-recognizable* and *co-Turing-recognizable*
  - Corollary: $\overline{A_{TM}}$ is *not* Turing-recognizable