The Church-Turing Thesis

CS60001: Foundations of Computing Science

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Turing Machines

- A Turing Machine is a 7-tuple, $(Q, \Sigma, \Gamma, \delta, q_0, q_{accept}, q_{reject})$, where
  - Q, Σ, Γ are all *finite* sets
  - Q is the set of *states*
  - Σ is the *input alphabet* not containing the *blank symbol* $\sqcup$
  - Γ is the stack alphabet, where $\sqcup \in \Gamma$ and $\Sigma \subseteq \Gamma$
  - $\delta: Q \times \Gamma \rightarrow Q \times \Gamma \times \{L, R\}$ is the *transition function*
  - $q_0 \in Q$ is the *start state*
  - $Q_{accept} \in Q$ is the *accept state*
  - $Q_{reject} \in Q$ is the *reject state*, where $Q_{reject} \neq Q_{accept}$

- Differences Between Finite Automata and Turing Machines
  - A Turing machine can both write on the tape and read from it
  - The read-write head can both move to the left and to the right
  - The tape is infinite
  - The special states for rejecting and accepting take effect immediately

Language for Turing Machines

- A Turing Machine *M* *accepts* input *w* if a sequence of configurations $C_1, C_2, \ldots, C_k$ exists, where
  - C_1 is the *start configuration* of *M* on input *w*
  - Each C_i *yields* C_{i+1}
  - C_k is an *accepting configuration*

- The collection of strings that *M* accepts is the *language of M*, or the *language recognized by M*, denoted by *L(M)*
  - A language is Turing-recognizable (recursively enumerable language) if some Turing machine recognizes it
  - A language is Turing-decidable (recursively language) or decidable if some Turing machine decides it
  - Every Turing-decidable language is also Turing-recognizable

Example of Turing Machine

Turing Machine *M* that decides *A* = $\{0^{2^n} \mid n \geq 0\}$

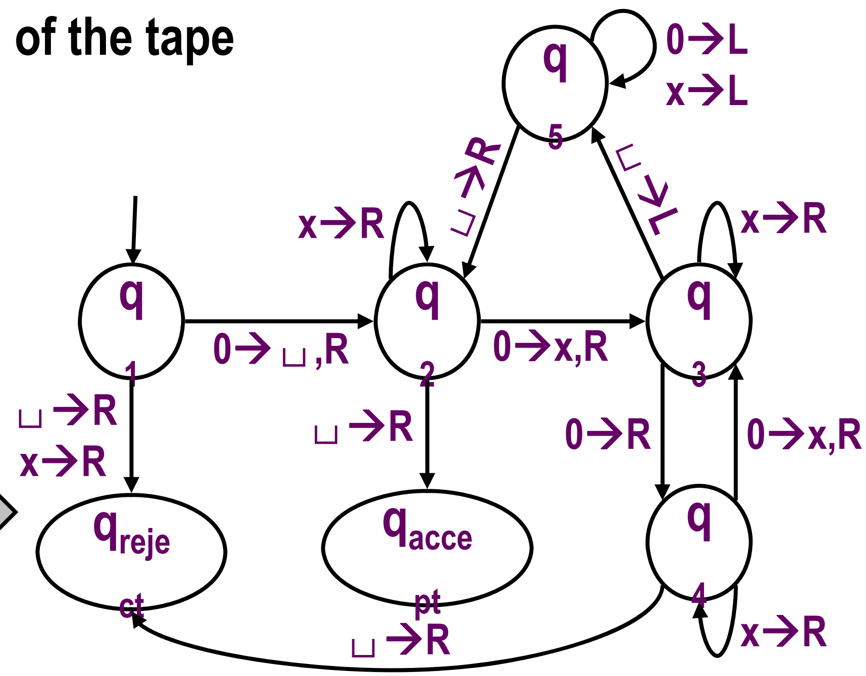
M on input string *w*,

1. Sweep left to right across the tape, crossing off every other 0
2. If in stage 1 the tape contained a single 0, *accept*
3. If in stage 1 the tape contained more than a single 0, and the number of 0s was odd, *reject*
4. Return the head to the left-hand end of the tape
5. Go to stage 1

$M = (Q, \Sigma, \Gamma, \delta, q_1, q_{accept}, q_{reject})$, where

- $Q = \{q_1, q_2, q_3, q_4, q_5, q_{accept}, q_{reject}\}$
- $q_1 \leftarrow$ start state
- $q_{accept} \leftarrow$ accept state
- $q_{reject} \leftarrow$ reject state
- $\Sigma = \{0\}$ and $\Gamma = \{0, x, \sqcup\}$

δ ⇒



Variants of Turing Machines

- **Multi-tape Turing Machines**
  - *Transition function*, $\delta: Q \times \Gamma^k \rightarrow Q \times \Gamma^k \times \{L, R, S\}^k$ (k is the number of tapes)
  - The expression $\delta(q_i, a_1, a_2, \ldots, a_k) = (q_j, b_1, b_2, \ldots, b_k, L, S, \ldots, L)$ means that,
    - the machine is in state q_i and heads *1* through *k* reading a_1 through a_k
    - the machine goes to state q_j, which symbols b_1 through b_k
    - the machine directs each head to move *left(L)*, *right(R)* or *stay put(S)*
  - Theorems:
    - Every multi-tape Turing machine has an equivalent single-tape Turing machine
    - A language is Turing-recognizable if and only if some multi-tape Turing machine recognizes it

Variants of Turing Machines (contd…)

- **Nondeterministic Turing Machines**
  - *Transition function*, $\delta: Q \times \Gamma \rightarrow \mathcal{P}\big(Q \times \Gamma \times \{L, R\}\big)$
  - Theorems:
    - Every nondeterministic Turing machine has an equivalent deterministic Turing machine
    - A language is Turing-recognizable if and only if some nondeterministic Turing machine recognizes it
    - A language is decidable if and only if some nondeterministic Turing machine decides it
- **Enumerators**
  - Starts with a blank input tape
  - Language can be infinite list of strings (may be repetitive and generated by the enumerator without halting)
  - Theorem: A language is Turing-recognizable if and only if some enumerator enumerates it

The Definition of Algorithm

- **Hilbert's 10th Problem:**
  - Devise an algorithm that tests whether a polynomial has an integral root.
  - D = {p | p is a polynomial with an integral root} – whether *D* is *decidable*
  - The answer is *negative*; in contrast *D* is *Turing-recognizable*

- **To Define Algorithms**
  - Church proposed notational system called λ-calculus
  - Turing proposed "machine"

  equivalent

- **The Church-Turing Thesis**

Intuitive notion of algorithms	equals	Turing machine algorithms