1. Prove revelation principle for BIC mechanisms.

2. Let $f : \Theta \rightarrow X$ be a social choice function such that we have the following for every $\theta \in \Theta$

$$\sum_{i=1}^{n} u_i(f(\theta), \theta_i) \geq \sum_{i=1}^{n} u_i(x, \theta_i) \forall x \in X$$

Show that $f$ is ex-post efficient.

3. (Taken from an exercise in [Nar14]) Let $N = \{1, 2\}, \Theta_1 = \{a_1, b_1\}, \Theta_2 = \{a_2, b_2\}, X = \{x, y, z\}$ and

- $u_1(x, a_1) = 100, u_1(y, a_1) = 50, u_1(z, a_1) = 0$
- $u_1(x, b_1) = 50, u_1(y, b_1) = 100, u_1(z, b_1) = 40$
- $u_2(x, a_2) = 0, u_2(y, a_2) = 50, u_2(z, a_2) = 100$
- $u_2(x, b_2) = 50, u_2(y, b_2) = 30, u_2(z, b_2) = 100$

For the above environment, give an example for a social choice function for each of the following cases (EPE: Ex-Post Efficient, DSIC: Dominant Strategy Incentive Compatible, BIC: Bayesian Incentive Compatible, D: Dictatorship, ND: Non-dictatorship).

(i) EPE, DSIC, and D
(ii) EPE, DSIC, and ND
(iii) Not EPE but DSIC and ND
(iv) EPE, BIC (under suitable prior), but not DSIC
(v) EPE but not BIC (under suitable prior)

4. Can a social choice function has more than one dictator if every player has a strict rational preference relation?

5. Consider the set of outcomes $X$ to be the set of integers in the range from 0 to 100. There are $n$ players. The type of player $i$ is $\theta_i$ and the utility of player $i$ is $u_i(x) = -|x - \theta_i|$ for every $x \in X$. Design $n$ social choice functions $f_1, \ldots, f_n : \times_{i \in [n]} \Theta_i \rightarrow X$ each one of which is non-dictatorship as well as DSIC.

References