CS60073: Advanced Machine Learning

Mid-semester Examination Spring 2019

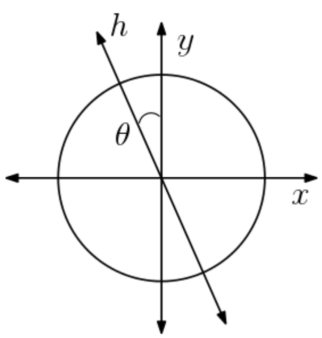
Answer all THREE questions. Answer all parts of a question together. Make suitable assumptions if necessary.

Time: 2 hrs. Total marks: 45.

---

1. We want to classify points in two-dimensional plane into two classes $\{+1, -1\}$. The (unknown) true label of a point (x, y) is given by $\mathrm{sign}(x)$. The input distribution $\mathcal{D}$ is the uniform distribution over the unit circle centered at the origin.

(a). Consider the hypothesis h as shown in the figure below (h classifies all the points on the right of the line as $+1$ and all the points to the left as -1). Compute the risk $L_{\mathcal{D}}(h)$, as a function of θ. Assume θ is in radians. [5]



(b). We obtain $\frac{1}{\theta}$ (which is given to be an integer ≥ 2) training examples (i.e., samples from $\mathcal{D}$, along with their true labels). What is the probability that we find at least an example that is misclassified by h? [5]

(c). Give an example of a distribution $\mathcal{D}$ under which h has risk zero. [5]

2 (a). Let $\mathcal{X} = \mathbb{R}^2$, $y = \{0, 1\}$, and $\mathcal{H}$ be the class of origin centered circles in the plane. That is, $\mathcal{H} = \{h_r : h_r(x) = 1, \text{ if } ||x|| \leq r, \text{ and } h_r(x) = 0, \text{ otherwise}\}$. Prove that $\mathcal{H}$ is PAC learnable (assume realizability), and its sample complexity is bounded by [10]

$$m_{\mathcal{H}}(\epsilon, \delta) \leq \left\lceil \frac{\log(1/\delta)}{\epsilon} \right\rceil.$$

(b). State when a hypothesis class $\mathcal{H}$ is said to have the *uniform convergence* property. [5]

3 (a). Given some finite domain set $\mathcal{X}$, and a number $k \leq |\mathcal{X}|$, find out the VC dimension for the class, $\mathcal{H}^{\mathcal{X}}_{=k} = \{h \in [0,1]^{\mathcal{X}} : |\{x : h(x) = 1\}| = k\}$, that is the set of all functions that assigns the value 1 to exactly k elements of $\mathcal{X}$. [10]

(b). Show that for a finite hypothesis class $\mathcal{H}$, $\mathrm{VCdim}(\mathcal{H}) \leq \log_2(|\mathcal{H}|)$. [5]