Chapter 16: Query Optimization

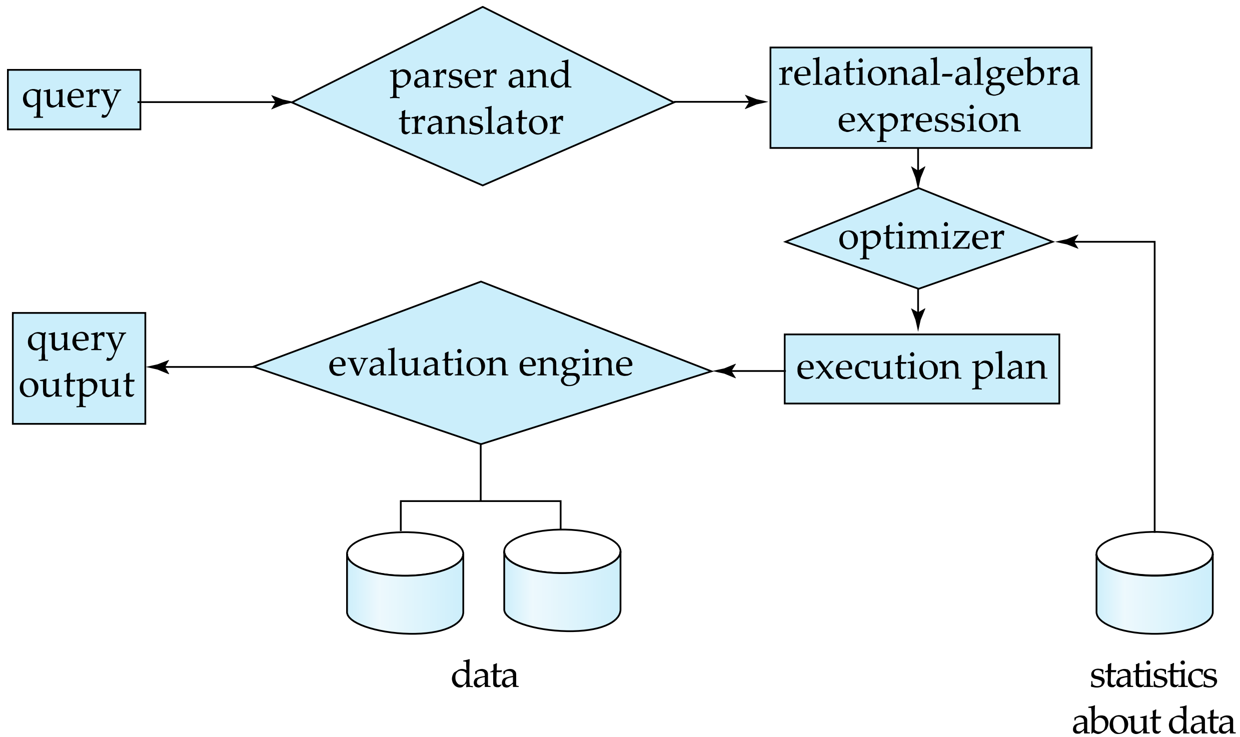
Database System Concepts, 7th Ed.

Outline

- Introduction
- Transformation of Relational Expressions
- Catalog Information for Cost Estimation
- Statistical Information for Cost Estimation
- Cost-based optimization
- Dynamic Programming for Choosing Evaluation Plans
- Materialized views

Basic Steps in Query Processing

1. Parsing and translation
2. Optimization
3. Evaluation



Introduction

- *Schema of an University:*

  instructor(*ID*, *name*, *dept name*, *salary*)

  teaches(*ID*, *course id*, *sec id*, *semester*, *year*)

  course(*course id*, *title*, *dept name*, *credits*)

Find the names of the instructors of Music department and the titles of the courses they taught.

- Alternative ways of evaluating a given query

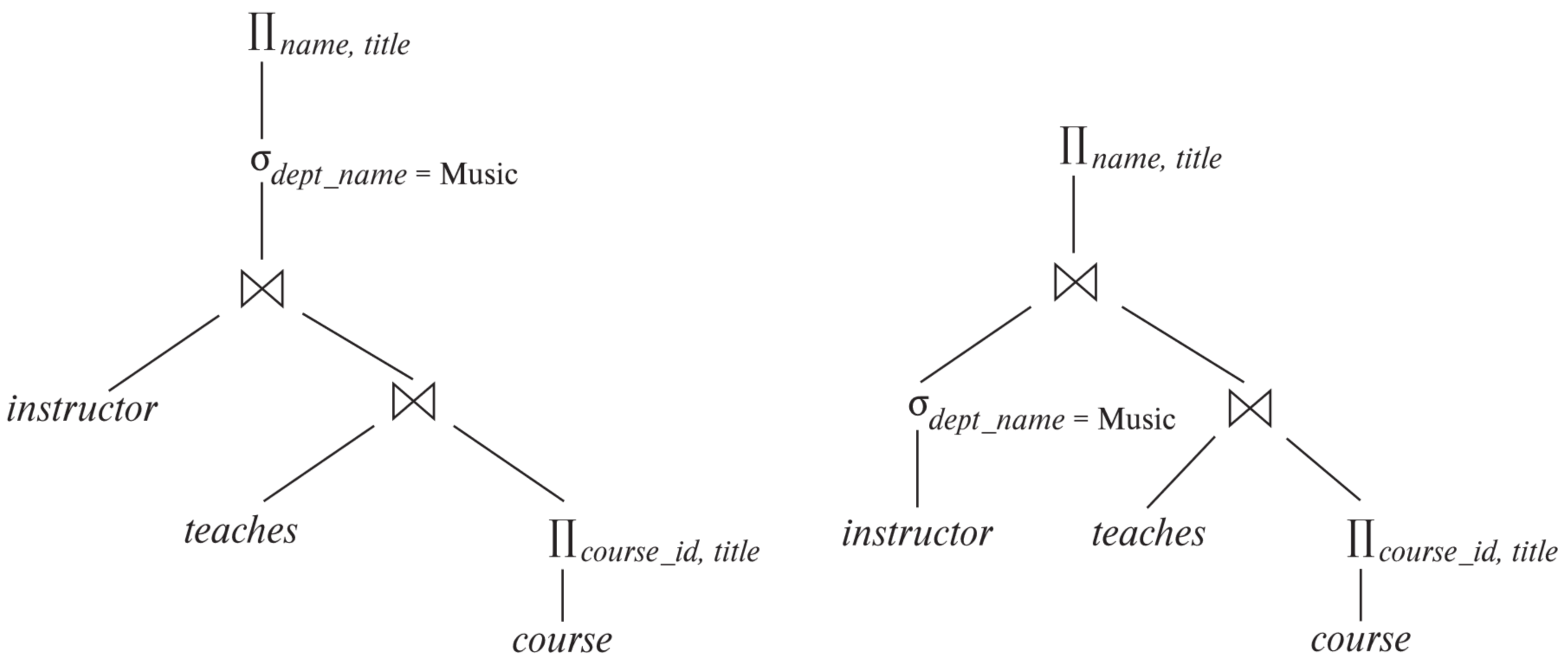
$\Pi_{name,title}\ (\sigma_{dept\ name\ =\text{“Music”}}\ (instructor \bowtie teaches \bowtie course))$

$\Pi_{name,title}\ (\sigma_{dept\ name\ =\text{“Music”}}\ (instructor \bowtie (teaches \bowtie \Pi_{course\ id,title}(course))))$

$\Pi_{name,title}\ ((\sigma_{dept\ name\ =\text{“Music”}}\ (instructor) \bowtie teaches) \bowtie \Pi_{course\ id,title}(course))$

Introduction

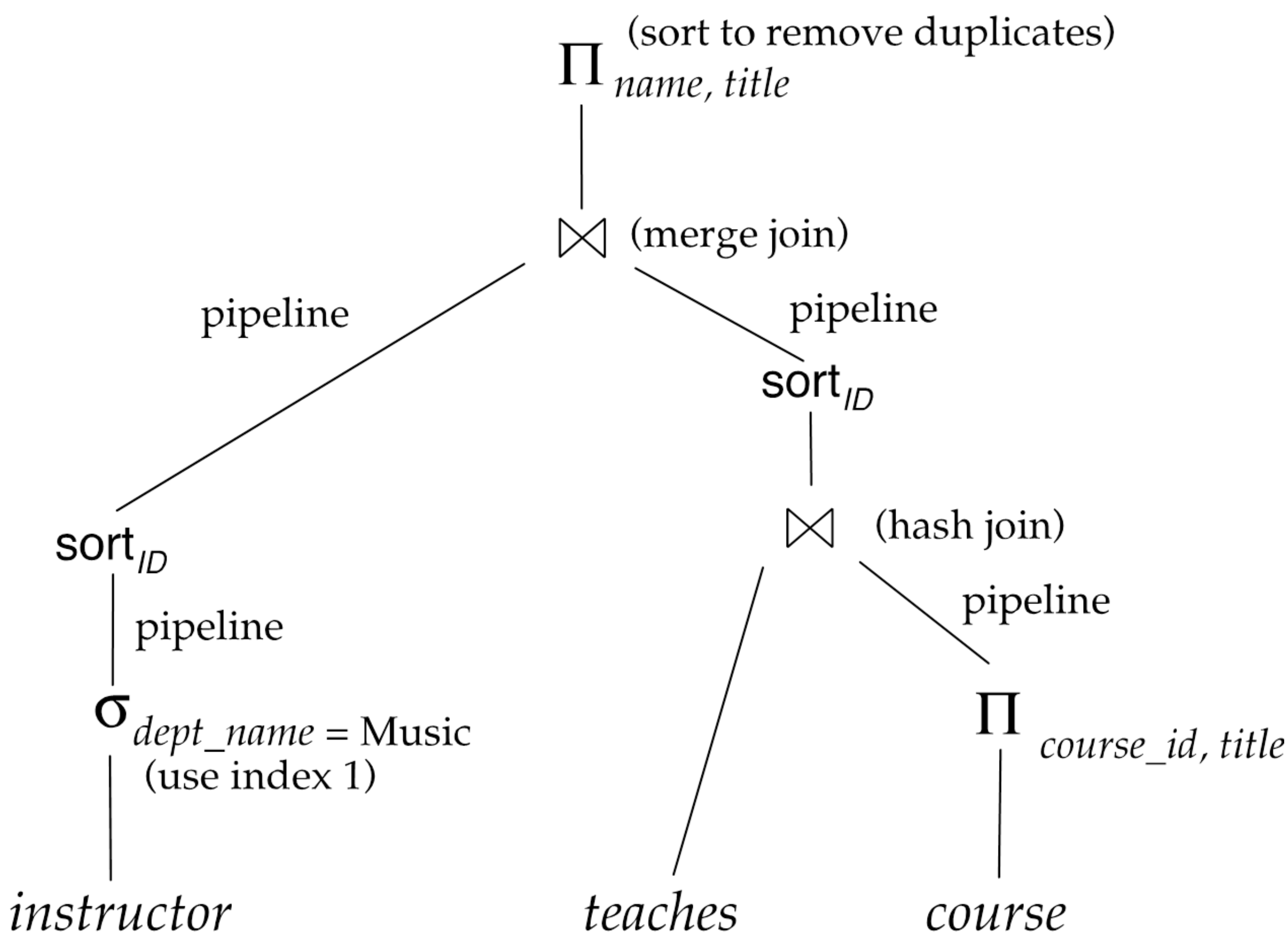
- Alternative ways of evaluating a given query
- $\Pi_{name,title}$ ($\sigma_{dept\ name}$ =“Music” ($instructor \bowtie (teaches \bowtie \Pi_{course\ id,title}(course))$))
  - Equivalent expressions
  - Different algorithms for each operation



(a) Initial expression tree (b) Transformed expression tree

Introduction (Cont.)

- An **evaluation plan** defines exactly what algorithm is used for each operation, and how the execution of the operations is coordinated.



Introduction (Cont.)

- Cost difference between evaluation plans for a query can be enormous
  - E.g., seconds vs. days in some cases
- Steps in **cost-based query optimization**
  1. Generate logically equivalent expressions using **equivalence rules**
  2. Annotate resultant expressions to get alternative query plans
  3. Choose the cheapest plan based on **estimated cost**
- Estimation of plan cost based on:
  - Statistical information about relations. Examples:
    - number of tuples, number of distinct values for an attribute
  - Statistics estimation for intermediate results
    - to compute cost of complex expressions
  - Cost formulae for algorithms, computed using statistics

Generating Equivalent Expressions

Transformation of Relational Expressions

- Two relational algebra expressions are said to be **equivalent** if the two expressions generate the same set of tuples on every *legal* database instance
  - Note: order of tuples is irrelevant
  - we don't care if they generate different results on databases that violate integrity constraints
- In SQL, inputs and outputs are multisets of tuples
  - Two expressions in the multiset version of the relational algebra are said to be equivalent if the two expressions generate the same multiset of tuples on every legal database instance.
- An **equivalence rule** says that expressions of two forms are equivalent
  - Can replace expression of first form by second, or vice versa

Equivalence Rules

1. Conjunctive selection operations can be deconstructed into a sequence of individual selections.
   $$\sigma_{\theta_1 \wedge \theta_2}(E) \equiv \sigma_{\theta_1}(\sigma_{\theta_2}(E))$$
2. Selection operations are commutative.
   $$\sigma_{\theta_1}(\sigma_{\theta_2}(E)) \equiv \sigma_{\theta_2}(\sigma_{\theta_1}(E))$$
3. Only the last in a sequence of projection operations is needed, the others can be omitted.
   $$\Pi_{L_1}(\Pi_{L_2}(\ldots(\Pi_{L_n}(E))\ldots)) \equiv \Pi_{L_1}(E)$$
   where $L_1 \subseteq L_2 \ldots \subseteq L_n$
4. Selections can be combined with Cartesian products and theta joins.
   a. $\sigma_{\theta}(E_1 \times E_2) \equiv E_1 \bowtie_{\theta} E_2$
   b. $\sigma_{\theta_1}(E_1 \bowtie_{\theta_2} E_2) \equiv E_1 \bowtie_{\theta_1 \wedge \theta_2} E_2$

Equivalence Rules (Cont.)

5. Theta-join operations (and natural joins) are commutative.

$$E_1 \bowtie E_2 \quad \equiv \quad E_2 \bowtie E_1$$

6. (a) Natural join operations are associative:

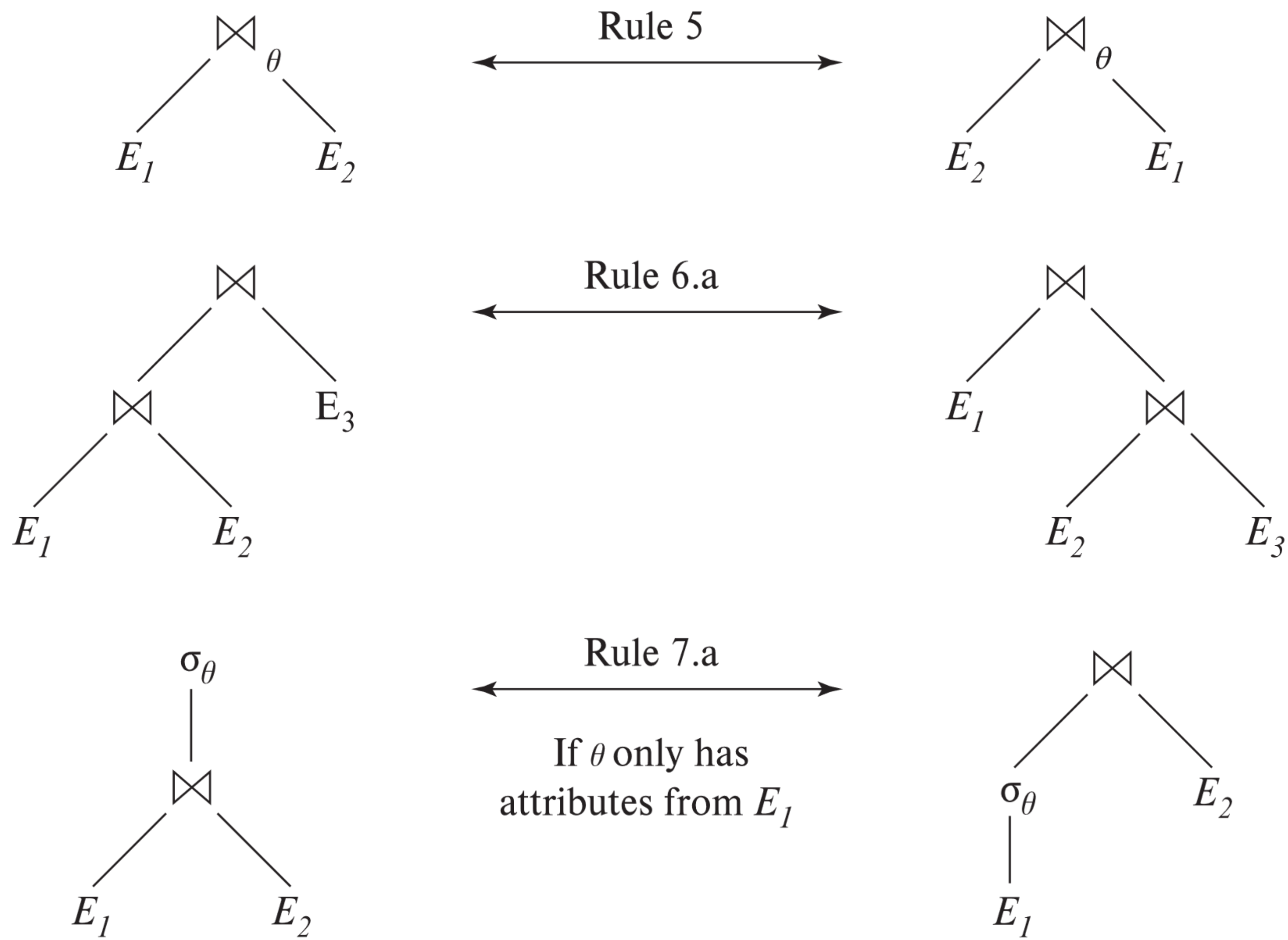
$$(E_1 \bowtie E_2) \bowtie E_3 \quad \equiv \quad E_1 \bowtie (E_2 \bowtie E_3)$$

(b) Theta joins are associative in the following manner:

$$(E_1 \bowtie_{\theta_1} E_2) \bowtie_{\theta_2 \wedge \theta_3} E_3 \quad \equiv \quad E_1 \bowtie_{\theta_1 \wedge \theta_3} (E_2 \bowtie_{\theta_2} E_3)$$

where θ_2 involves attributes from only E_2 and E_3.

Pictorial Depiction of Equivalence Rules



Equivalence Rules (Cont.)

7. The selection operation distributes over the theta join operation under the following two conditions:

   (a) When all the attributes in θ_0 involve only the attributes of one of the expressions (E_1) being joined.

   $$\sigma_{\theta_0}(E_1 \bowtie_\theta E_2) \equiv (\sigma_{\theta_0}(E_1)) \bowtie_\theta E_2$$

   (b) When θ_1 involves only the attributes of E_1 and θ_2 involves only the attributes of E_2.

   $$\sigma_{\theta_1 \wedge \theta_2}(E_1 \bowtie_\theta E_2) \equiv (\sigma_{\theta_1}(E_1)) \bowtie_\theta (\sigma_{\theta_2}(E_2))$$

Equivalence Rules (Cont.)

8. The projection operation distributes over the theta join operation as follows:

   (a) if θ involves only attributes from $L_1 \cup L_2$:

   $$\Pi_{L_1 \cup L_2}(E_1 \bowtie_\theta E_2) \equiv \Pi_{L_1}(E_1) \bowtie_\theta \Pi_{L_2}(E_2)$$

   (b) In general, consider a join $E_1 \bowtie_\theta E_2$.

   - Let L_1 and L_2 be sets of attributes from E_1 and E_2, respectively.
   - Let L_3 be attributes of E_1 that are involved in join condition θ, but are not in $L_1 \cup L_2$, and
   - let L_4 be attributes of E_2 that are involved in join condition θ, but are not in $L_1 \cup L_2$.

     $$\Pi_{L_1 \cup L_2}(E_1 \bowtie_\theta E_2) \equiv \Pi_{L_1 \cup L_2}(\Pi_{L_1 \cup L_3}(E_1) \bowtie_\theta \Pi_{L_2 \cup L_4}(E_2))$$

Similar equivalences hold for outerjoin operations: ⟕, ⟖, and ⟗

Equivalence Rules (Cont.)

9. The set operations union and intersection are commutative

$$E_1 \cup E_2 \equiv E_2 \cup E_1$$
$$E_1 \cap E_2 \equiv E_2 \cap E_1$$

(set difference is not commutative).

10. Set union and intersection are associative.

$$(E_1 \cup E_2) \cup E_3 \equiv E_1 \cup (E_2 \cup E_3)$$
$$(E_1 \cap E_2) \cap E_3 \equiv E_1 \cap (E_2 \cap E_3)$$

11. The selection operation distributes over $\cup$, $\cap$ and –.

a. $\sigma_\theta (E_1 \cup E_2) \equiv \sigma_\theta (E_1) \cup \sigma_\theta(E_2)$

b. $\sigma_\theta (E_1 \cap E_2) \equiv \sigma_\theta (E_1) \cap \sigma_\theta(E_2)$

c. $\sigma_\theta (E_1 - E_2) \equiv \sigma_\theta (E_1) - \sigma_\theta(E_2)$

d. $\sigma_\theta (E_1 \cap E_2) \equiv \sigma_\theta(E_1) \cap E_2$

e. $\sigma_\theta (E_1 - E_2) \equiv \sigma_\theta(E_1) - E_2$

preceding equivalence does not hold for $\cup$

12. The projection operation distributes over union

$$\Pi_L(E_1 \cup E_2) \equiv (\Pi_L(E_1)) \cup (\Pi_L(E_2))$$

Equivalence Rules (Cont.)

13. Selection distributes over aggregation as below

    $\sigma_\theta({}_G\gamma_A(E)) \equiv {}_G\gamma_A(\sigma_\theta(E))$

    provided θ only involves attributes in G

14. a. Full outerjoin is commutative:

    $E_1 ⟗ E_2 \equiv E_2 ⟗ E_1$

    b. Left and right outerjoin are not commutative, but:

    $E_1 ⟕ E_2 \equiv E_2 ⟖ E_1$

15. Selection distributes over left and right outerjoins as below, provided θ_1 only involves attributes of E_1

    a. $\sigma_{\theta_1}(E_1 ⟕_\theta E_2) \equiv (\sigma_{\theta_1}(E_1)) ⟕_\theta E_2$

    b. $\sigma_{\theta_1}(E_1 ⟖_\theta E_2) \equiv E_2 ⟕_\theta (\sigma_{\theta_1}(E_1))$

16. Outerjoins can be replaced by inner joins under some conditions

    a. $\sigma_{\theta_1}(E_1 ⟕_\theta E_2) \equiv \sigma_{\theta_1}(E_1 \bowtie_\theta E_2)$

    b. $\sigma_{\theta_1}(E_1 ⟖_\theta E_1) \equiv \sigma_{\theta_1}(E_1 \bowtie_\theta E_2)$

    provided θ_1 is null rejecting on E_2

Equivalence Rules (Cont.)

Note that several equivalences that hold for joins do not hold for outerjoins

- $\sigma_{\text{year=2017}}(\textit{instructor} ⟕ \textit{teaches}) \not\equiv \textit{instructor} ⟕ \sigma_{\text{year=2017}}(\textit{teaches})$

However in some cases outer and inner joins are equivalent

- $\sigma_{\text{year=2017}}(\textit{instructor} ⟕ \textit{teaches}) \equiv \sigma_{\text{year=2017}}(\textit{instructor} \bowtie \textit{teaches})$

- Outerjoins are not associative

$$(r ⟕ s) ⟕ t \quad \not\equiv \quad r ⟕ (s ⟕ t)$$

Transformation Example: Pushing Selections

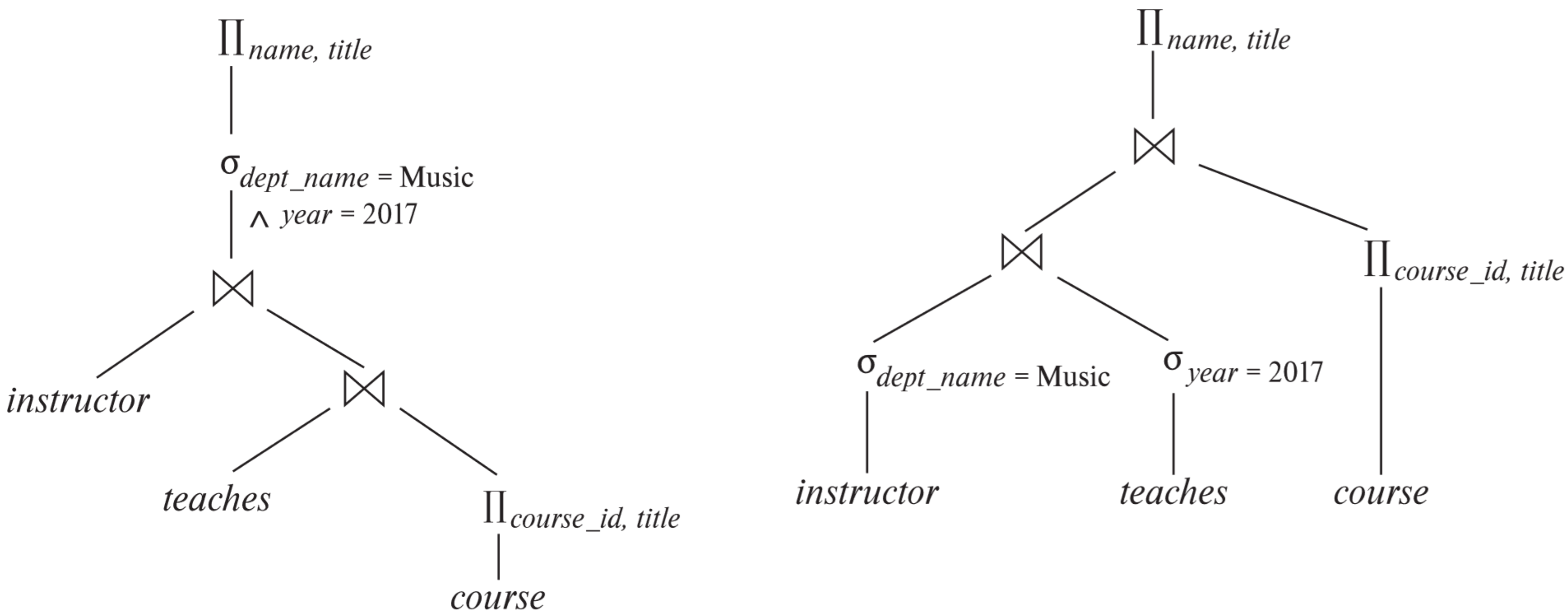
- Query: Find the names of all instructors in the Music department, along with the titles of the courses that they teach
  - $\Pi_{name,\ title}(\sigma_{dept\_name=\text{'Music'}}(instructor \bowtie (teaches \bowtie \Pi_{course\_id,\ title}(course))))$
- Transformation using rule 7a.
  - $\Pi_{name,\ title}((\sigma_{dept\_name=\text{'Music'}}(instructor)) \bowtie (teaches \bowtie \Pi_{course\_id,\ title}(course)))$
- Performing the selection as early as possible reduces the size of the relation to be joined.

Example with Multiple Transformations

- Query: Find the names of all instructors in the Music department who have taught a course in 2017, along with the titles of the courses that they taught
  - $\Pi_{name,\ title}(\sigma_{dept\_name=\text{"Music"} \wedge year = 2017}$
    $(instructor \bowtie (teaches \bowtie \Pi_{course\_id,\ title}(course))))$
- Transformation using join associatively (Rule 6a):
  - $\Pi_{name,\ title}(\sigma_{dept\_name=\text{"Music"} \wedge year = 2017}$
    $((instructor \bowtie teaches) \bowtie \Pi_{course\_id,\ title}(course)))$
- Second form provides an opportunity to apply the "perform selections early" rule, resulting in the subexpression

  $\sigma_{dept\_name = \text{"Music"}}(instructor) \bowtie \sigma_{year = 2017}(teaches)$

Multiple Transformations (Cont.)



(a) Initial expression tree

(b) Tree after multiple transformations

Transformation Example: Pushing Projections

- Consider: $\Pi_{name,\ title}(\sigma_{dept\_name=\text{"Music"}}(instructor) \bowtie teaches)$
  $\bowtie \Pi_{course\_id,\ title}(course))))$
- When we compute

  $(\sigma_{dept\_name = \text{"Music"}}(instructor \bowtie teaches)$

  we obtain a relation whose schema is:
  (*ID, name, dept_name, salary, course_id, sec_id, semester, year)*
- Push projections using equivalence rules 8a and 8b; eliminate unneeded attributes from intermediate results to get:

  $\Pi_{name,\ title}(\Pi_{name,\ course\_id}($
  $\sigma_{dept\_name=\text{"Music"}}(instructor) \bowtie teaches))$
  $\bowtie \Pi_{course\_id,\ title}(course))))$
- Performing the projection as early as possible reduces the size of the relation to be joined.

Join Ordering Example

- For all relations r_1, r_2, and r_3,

$$(r_1 \bowtie r_2) \bowtie r_3 = r_1 \bowtie (r_2 \bowtie r_3)$$

  (Join Associativity) $\bowtie$

- If $r_2 \bowtie r_3$ is quite large and $r_1 \bowtie r_2$ is small, we choose

$$(r_1 \bowtie r_2) \bowtie r_3$$

  so that we compute and store a smaller temporary relation.

Join Ordering Example (Cont.)

- Consider the expression

$$\Pi_{name,\ title}(\sigma_{dept\_name=\text{"Music"}}(instructor) \bowtie teaches)$$
$$\bowtie \Pi_{course\_id,\ title}(course))))$$

- Could compute $teaches \bowtie \Pi_{course\_id,\ title}(course)$ first, and join result with
  $$\sigma_{dept\_name=\text{"Music"}}(instructor)$$
  but the result of the first join is likely to be a large relation.
- Only a small fraction of the university's instructors are likely to be from the Music department
  - it is better to compute
    $$\sigma_{dept\_name=\text{"Music"}}(instructor) \bowtie teaches$$
    first.

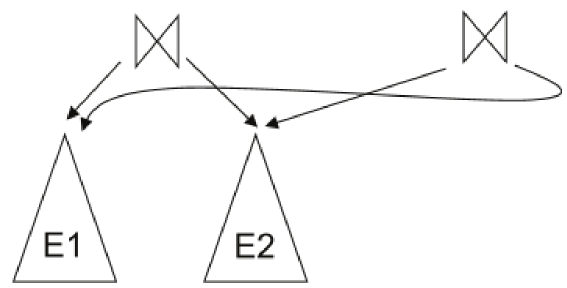
Enumeration of Equivalent Expressions

- Query optimizers use equivalence rules to **systematically** generate expressions equivalent to the given expression
- Can generate all equivalent expressions as follows:
  - Repeat
    - apply all applicable equivalence rules on every subexpression of every equivalent expression found so far
    - add newly generated expressions to the set of equivalent expressions

    Until no new equivalent expressions are generated above
- The above approach is very expensive in space and time
  - Two approaches
    - Optimized plan generation based on transformation rules
    - Special case approach for queries with only selections, projections and joins

Implementing Transformation Based Optimization

- Space requirements reduced by sharing common sub-expressions:
  - when E1 is generated from E2 by an equivalence rule, usually only the top level of the two are different, subtrees below are the same and can be shared using pointers
    - E.g., when applying join commutativity



  - Same sub-expression may get generated multiple times
    - Detect duplicate sub-expressions and share one copy
- Time requirements are reduced by not generating all expressions
  - Dynamic programming
    - We will study only the special case of dynamic programming for join order optimization

Choice of Evaluation Plans

- Must consider the interaction of evaluation techniques when choosing evaluation plans
  - choosing the cheapest algorithm for each operation independently may not yield best overall algorithm. E.g.
    - merge-join may be costlier than hash-join, but may provide a sorted output which reduces the cost for an outer level aggregation.
    - nested-loop join may provide opportunity for pipelining
- Practical query optimizers incorporate elements of the following two broad approaches:
  1. Search all the plans and choose the best plan in a cost-based fashion.
  2. Uses heuristics to choose a plan.

Cost-Based Optimization

- Consider finding the best join-order for $r_1 \bowtie r_2 \bowtie \ldots \bowtie r_n$.
- There are $(2(n - 1))!/(n - 1)!$ different join orders for above expression. With $n = 7$, the number is 665280, with $n = 10$, the number is greater than 176 billion!
- No need to generate all the join orders. Using dynamic programming, the least-cost join order for any subset of
  $\{r_1, r_2, \ldots r_n\}$ is computed only once and stored for future use.

Dynamic Programming in Optimization

- To find best join tree for a set of *n* relations:
  - To find best plan for a set *S* of *n* relations, consider all possible plans of the form: $S_1 \bowtie (S - S_1)$ where S_1 is any non-empty subset of *S*.
  - Recursively compute costs for joining subsets of *S* to find the cost of each plan. Choose the cheapest of the $2^n - 2$ alternatives.
  - Base case for recursion: single relation access plan
    - Apply all selections on R_i using best choice of indices on R_i
  - When plan for any subset is computed, store it and reuse it when it is required again, instead of recomputing it
    - Dynamic programming

Heuristic Optimization

- Cost-based optimization is expensive, even with dynamic programming.
- Systems may use *heuristics* to reduce the number of choices that must be made in a cost-based fashion.
- Heuristic optimization transforms the query-tree by using a set of rules that typically (but not in all cases) improve execution performance:
  - Perform selection early (reduces the number of tuples)
  - Perform projection early (reduces the number of attributes)
  - Perform most restrictive selection and join operations (i.e., with smallest result size) before other similar operations.
  - Some systems use only heuristics, others combine heuristics with partial cost-based optimization.

Statistics for Cost Estimation

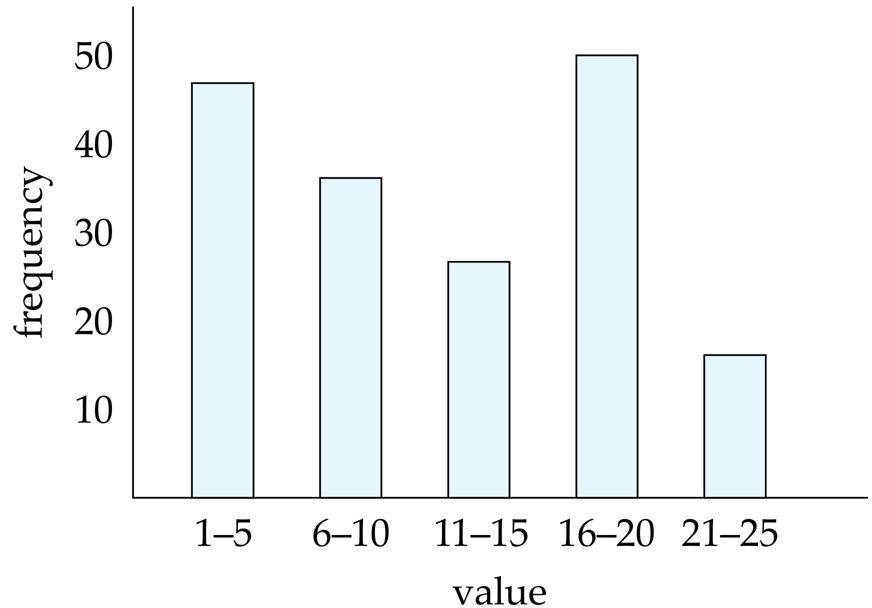
Statistical Information for Cost Estimation

- n_r: number of tuples in a relation *r.*
- b_r: number of blocks containing tuples of *r.*
- l_r: size of a tuple of *r.*
- f_r: blocking factor of *r* — i.e., the number of tuples of *r* that fit into one block.
- *V(A, r):* number of distinct values that appear in *r* for attribute *A;* same as the size of $\prod_A(r)$.
- If tuples of *r* are stored together physically in a file, then:

$$b_r = \left\lceil \frac{n_r}{f_r} \right\rceil$$

Histograms

- Histogram on attribute *age* of relation *person*



- **Equi-width** histograms
- **Equi-depth** histograms break up range such that each range has (approximately) the same number of tuples
  - E.g. (4, 8, 14, 19)
- Many databases also store *n* **most-frequent values** and their counts
  - Histogram is built on remaining values only

Selection Size Estimation

- $\sigma_{A=v}(r)$
  - $n_r / V(A,r)$: number of records that will satisfy the selection
  - Equality condition on a key attribute: *size estimate* = 1
- $\sigma_{A \leq V}(r)$ (case of $\sigma_{A \geq V}(r)$ is symmetric)
  - Let c denote the estimated number of tuples satisfying the condition.
  - If min(A,r) and max(A,r) are available in catalog
    - c = 0 if v < min(A,r)
    - c = $n_r . \dfrac{v - \min(A,r)}{\max(A,r) - \min(A,r)}$
  - If histograms available, can refine above estimate
  - In absence of statistical information *c* is assumed to be $n_r / 2$.

Size Estimation of Complex Selections

- The **selectivity** of a condition θ_i is the probability that a tuple in the relation *r* satisfies θ_i .
  - If s_i is the number of satisfying tuples in *r*, the selectivity of θ_i is given by s_i / n_r.
- **Conjunction:** $\sigma_{\theta 1 \wedge \theta 2 \wedge \ldots \wedge \theta n}(r)$. *Assuming independence,* estimate of tuples in the result is:
$$n_r * \frac{s_1 * s_2 * \ldots * s_n}{n_r^n}$$
- **Disjunction:** $\sigma_{\theta 1 \vee \theta 2 \vee \ldots \vee \theta n}(r)$. Estimated number of tuples:
$$n_r * \left(1 - (1 - \frac{s_1}{n_r}) * (1 - \frac{s_2}{n_r}) * \ldots * (1 - \frac{s_n}{n_r})\right)$$
- **Negation:** $\sigma_{\neg\theta}(r)$. Estimated number of tuples:
$$n_r - size(\sigma_\theta(r))$$

Join Operation: Running Example

Running example:

student ⋈ *takes*

Catalog information for join examples:

- $n_{student}$ = 5,000.
- $f_{student}$ = *50,* which implies that
  $b_{student}$ =5000/50 = 100.
- n_{takes} = 10000.
- f_{takes} = 25, which implies that
  b_{takes} = 10000/25 = 400.
- *V(ID, takes)* = 2500, which implies that on average, each student who has taken a course has taken 4 courses.
  - Attribute *ID* in *takes* is a foreign key referencing *student.*
  - *V*(*ID, student*) = 5000 (*primary key!*)

Estimation of the Size of Joins

- The Cartesian product $r \times s$ contains $n_r . n_s$ tuples; each tuple occupies $s_r + s_s$ bytes.
- If $R \cap S = \emptyset$, then $r \bowtie s$ is the same as $r \times s$.
- If $R \cap S$ is a key for R, then a tuple of s will join with at most one tuple from r
  - therefore, the number of tuples in $r \bowtie s$ is no greater than the number of tuples in s.
- If $R \cap S$ *in* S is a foreign key in S referencing R, then the number of tuples in $r \bowtie s$ is exactly the same as the number of tuples in s.
  - The case for $R \cap S$ being a foreign key referencing S is symmetric.
- In the example query *student* $\bowtie$ *takes, ID* in *takes* is a foreign key referencing *student*
  - hence, the result has exactly n_{takes} tuples, which is 10000

Estimation of the Size of Joins (Cont.)

- If $R \cap S = \{A\}$ is not a key for R or S.
  If we assume that every tuple t in R produces tuples in $R \bowtie S$, the number of tuples in $R \bowtie S$ is estimated to be:

  $$\frac{n_r * n_s}{V(A,s)}$$

  If the reverse is true, the estimate obtained will be:

  $$\frac{n_r * n_s}{V(A,r)}$$

  The lower of these two estimates is probably the more accurate one.
- Can improve on above if histograms are available
  - Use formula similar to above, for each cell of histograms on the two relations

Estimation of the Size of Joins (Cont.)

- Compute the size estimates for *student* ⋈ *takes* without using information about foreign keys:
  - *V(ID, takes)* = 2500, and
    V(ID, student) = 5000
  - The two estimates are 5000 * 10000/2500 = 20,000 and 5000 * 10000/5000 = 10000
  - We choose the lower estimate, which in this case, is the same as our earlier computation using foreign keys.

Size Estimation for Other Operations

- Projection: estimated size of $\prod_A(r) = V(A,r)$
- Aggregation : estimated size of ${}_G\gamma_A(r) = V(G,r)$
- Set operations
  - For unions/intersections of selections on the same relation: rewrite and use size estimate for selections
    - E.g., $\sigma_{\theta 1}(r) \cup \sigma_{\theta 2}(r)$ can be rewritten as $\sigma_{\theta 1 \text{ or } \theta 2}(r)$
  - For operations on different relations:
    - estimated size of $r \cup s$ = size of r + size of s.
    - estimated size of $r \cap s$ = minimum size of r and size of s.
    - estimated size of $r - s = r$.
    - All the three estimates may be quite inaccurate, but provide upper bounds on the sizes.

Size Estimation (Cont.)

- Outer join:
  - Estimated size of $r ⟕ s$ = *size of* $r \bowtie s$ + *size of* r
    - Case of right outer join is symmetric
  - Estimated size of $r ⟗ s$ = *size of* $r \bowtie s$ + size of r + size of s

End of Chapter