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RMO WORKSHOP THEORY

COMBINATORICS

- **Addition Principle:**
  Number of ways of choosing an element from A or(exclusive) B is |A|+|B| where |A| is number of elements in set A (Cardinality) and |B| is number of elements in set |B|.

- **Multiplication Principle:**
  Number of ways of choosing an element from A and B is |A|.|B| where |A| is number of elements in set A and |B| is number of elements in set |B|.

- **Inclusion Exclusion Principle:**

$$\left|\bigcup_{i=1}^{n} A_i\right| = \sum_{k=1}^{n}(-1)^{k+1}\left(\sum_{1\le i_1<......<i_k\le n}\left|\bigcap_{j=1}^{k} A_{i_j}\right|\right)$$

  where $A_1, A_2.....A_n$ are finite sets.

- To get **Complementary form of Inclusion Exclusion Principle**:

$$\left|\bigcap_{i=1}^{n}\bar{A}_i\right| = \left|S-\bigcup_{i=1}^{n} A_i\right|$$

- **Pascal's Identity:**

$$\binom{n+1}{k+1} = \binom{n}{k+1} + \binom{n}{k}$$

- **Binomial Theorem:**

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^k y^{n-k}$$

Here n is a natural number. Note Binomial Theorem can be extended if n is real.

- **Multinomial Theorem:**

$$\left(\sum_{i=1}^{m} x_i\right)^n = \sum_{k_1+k_2+....k_m=n} \binom{n}{k_1, k_2....., k_m} \prod_{1\le t\le m} x_t^{k_t}$$

- **Stars and Walls:**

The number of solutions for $\sum_{i=1}^{n} a_i = r$ where all a_i's are whole numbers is

$$\binom{n+r-1}{n-1}$$

PROBLEMS

- Let X={1,2,..10} .Find the number of pairs {A,B} such that A and B are subsets of X , $A \neq B, A \cup B = \{2,3,5,7\}$.

- Prove that $\sum_{i=0}^{n} \binom{n}{i}^2 = \binom{2n}{n}$

- Find the number of derangements on a set of cardinality 5. (A derangement is defined as a permutation of the elements of a set such that none of the elements appear on their original permutation)

- Find the number of 4 digit numbers in base 10 such that they have all non-zero digits and are divisible by 4 but not by 8.

- Find number of all 6 digit natural numbers such that the sum of their digits is 10 and each of the digits 0,1,2,3 occurs at least once.

- Find the number of tuples (A,B,C) where A,B and C are subsets of the set {1,2,….,n} satisfying $A \cap B \cap C = \phi, A \cap B \neq \phi, B \cap C \neq 0$.

BOOKS FOR REFERENCE:

Combinatorics by Chong/Meng

Applied Combinatorics by Alan Tucker