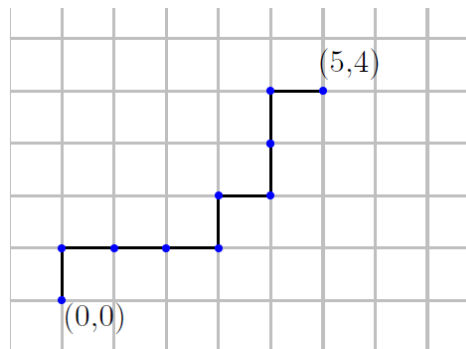


COMBINATORICS

- For any pair of **natural numbers** n and k , find the number of distinct n -tuples of **non-negative** integers whose sum is k .
- Prove that in a party of 6 people there are either three mutual strangers or three mutual acquaintances.
- How many routes are there from the point $(0, 0)$ to the point $(5, 4)$, if your step in the path is either one grid unit to the right or one unit upwards? The example below shows such a path.



- Binomial Identities:
$$\sum_{k=0}^n \binom{n}{k} k = n(2)^{n-1}$$

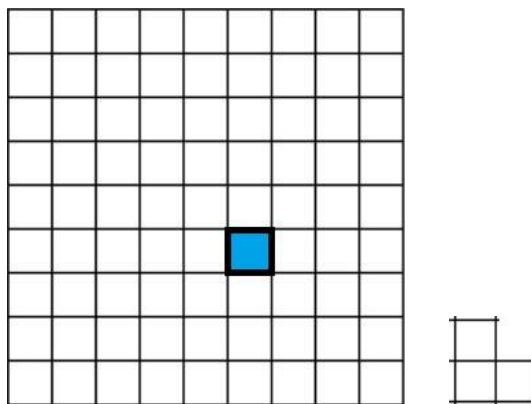
$$\sum_{k=0}^n \binom{n}{k} k^2 = (n+n^2)(2)^{n-2}$$
- Vandermonde
$$\sum_{i=0}^k \binom{m}{i} \binom{n}{k-i} = \binom{m+n}{k}$$
- Pascal
$$\binom{n}{i} + \binom{n}{i-1} = \binom{n+1}{i}$$
- Extended Pascal
$$\sum_{m=0}^n \binom{m}{k} = \binom{n+1}{k+1}$$

Easy problems:

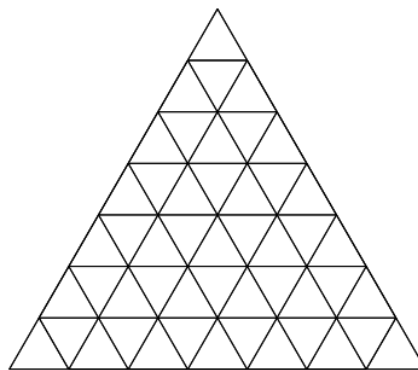
- For any pair of **natural numbers** n and k , find the number of distinct n -tuples of **positive** integers whose sum is k .
- Let $p+q+r=n$ where all p,q,r,n are natural numbers. Find number of Ordered (p,q,r) such that they form sides of a triangle.
- Consider third problem (on first page) .
Let A be point $(2,3)$ and B be $(4,2)$ on same figure.
Find the number of paths from $(0,0)$ to $(5,4)$ if :
 - Path passes through A
 - Path cannot pass through B
 - Path passes through A but not B

Moderate Problems

- Prove that it is possible to tile an $2^n \times 2^n$ board with any one tile removed by using a domino tile.



- Find number of triangles with only one vertex facing upward and sides of the triangle are parallel to the sides of the outside triangle. The side of main triangle is of integral length n .



Bonus: Hard Problems!

- Prove that in Problem 2(page 1) we can actually find at least two such groups of three people who are either strangers or acquaintances mutually.
- Find the number of paths from $(0,0)$ to (n,n) having property that all paths lie below or touch the diagonal line but don't cross that.

BOOK FOR REFERENCE: Combinatorics by Chong

FEEL FREE TO EMAIL ME FOR DOUBTS AND INFORM ME YOUR RMO RESULTS!