Problem Set :- MIHIR .

1] $n^4 + 4^n$ is always composite $n \geq 2$ $n \in \mathbb{N}$

2] 5 lattice points are chosen in the plane.
PT you can always choose two of these points such that segment joining through these two points passes through another lattice point.

3] Let $a, b, c \in \mathbb{Z}$ [distinct]. Let $P(x)$ be polynomial with integer ~~cour~~-efficients.
PT $P(a) = b$ $P(b) = c$ $P(c) = a$ all cannot be true.

4] Solve the system $x^5 + y^5 = 33$ $x + y = 5$.

5] Solve the equation $\sqrt[4]{97 - x} + \sqrt[4]{x} = 5$.

6] $a, b, c \in \mathbb{R}^+$ s.t $a^3 + b^3 = c^3$.
PT $a^2 + b^2 - c^2 > 6(c-a)(c-b)$.

7] In a finite sequence of real numbers every 7 sum is negative, whereas every 11 sum is positive. Find the greatest number of terms in such a sequence.

8] In how many parts will n lines, all intersecting each other, no three concurrent divide the plane ?

9] There are n points in plane. Any three of the points form a triangle of area ≤ 1. PT all n points lie inside a triangle of area ≤ 4

10] All points with integer co-ordinates in X-Y plane are coloured using three colours R, B & G. each colour used atleast once. Point (0,0) is coloured R and (0,1) is B. PT there exist 3 points with integer co-ordinates of diff. colours which form vertices of right angled triangle.

11] There is no quadruple of positive integers (x, y, z, u) Satisfying $x^2+y^2=3(z^2+u^2)$.

12] All points with integer co-ordinates are coloured using three colours PT We can find a monochromatic triangle which is either isosceles or its angles are in GP.

13]. A chess grandmaster is preparing for a tournament. There are 77 days for the tournament and he will play atleast one game everyday. In total he has decided to play 132 games. PT there will exist a succession of days in which he has played exactly 21 games.

14] Show that a 10x10 board cannot be covered by 25 straight tetrominoes.

 ← Tetromino.