

**School of Mathematical and Computational Sciences**  
**Indian Association for the Cultivation of Science**

*Master's/Integrated Master's-PhD Program/ Integrated  
Bachelor's-Master's Program/PhD Course*

**Theory of Computation II: COM 5108**

*Tutorial IX (09 November 2023)*

*Instructor: Goutam Biswas*

*Autumn Semester 2023*

1. Express  $a \vee b = c \wedge d$  in equivalent 3CNF form.
2. Show that if  $\mathbf{NP} = \mathbf{P}^{SAT}$ , then  $\mathbf{NP} = \mathbf{coNP}$ .
3. Let  $MAX-INDSET = \{ \langle G, k \rangle : \text{the size of the largest independent set of } G \text{ is } k \}$   
Show that  $MAX-INDSET \in \mathbf{P}^{SAT}$ .
4. Answer the following questions:
  - (a) Show that there a language  $L \in DTIME(2^{2^n})$  but  $L \notin DTIME(2^{n^k})$ , where  $k$  is a constant.
  - (b) For a language  $L$  we define  $U_L = \{1^n : \text{binary representation of } n \text{ is in } L\}$ .  
Show that  $L$  is decidable if and only if  $U_L$  is decidable.
  - (c) What is the time complexity to decide  $U_L$  when  $L \in DTIME(2^{2^n})$ ?
  - (d) Is there a decidable language in  $\mathbf{P/poly}$  that is not in  $\mathbf{P}$ ?
5. A language  $L \subseteq \{0, 1\}^*$  is *sparse* if there is a polynomial  $p$  such that  $|\{0, 1\}^n \cap L| \leq p(n)$  for all  $n \in \mathbb{N}_0$ .  
Prove that every sparse language is in  $\mathbf{P/poly}$ .