School of Mathematical and Computational Sciences

Indian Association for the Cultivation of Science

Master's/Integrated Master's-PhD Program/ Integrated Bachelor's-Master's Program/PhD Course

Theory of Computation II: COM 5108

Tutorial VII (19 October 2023)

Instructor: Goutam Biswas *Autumn Semester 2023*

1. Justify that $\mathbf{NL} \subseteq \mathbf{P}$.
2. We defined SAT_H as follows:

$$SAT_H = \{\phi 01^{n^{H(n)}} : \phi \in SAT, |\phi| = n\}.$$

The function $H(n)$ is defined as follows:

$$H(n) = \begin{cases} i, & i = \min\{j : j \text{ satisfies } C\} \\ \log\log n, & \text{otherwise.} \end{cases}$$

C: j is a natural number, $1 \leq j < \log\log n$, such that the TM M_j decides the membership of all $x \in \{0,1\}^*$, $|x| \leq \log n$ in SAT_H, within $j \times |x|^j$ steps,

   (a) Is $H(n)$ non-decreasing?
   (b) What is the maximum number of TMs to simulate to compute $H(n)$?
   (c) On how many input each machine runs?
   (d) What is the upper bound of time to check the membership of ϕ in SAT for all input?
   (e) Give an upper bound of computation time of $H(n)$.

3. Let $PAL = \{x \in \{0,1\}^* : x = x^R\}$. Show that $PAL \in \mathbf{L}$.
4. Let x be the binary representation of a positive integer (without leading zeros). The function f computes $f(x) = x + x^R$, where x^R is reverse of x. Give a logspace algorithm to compute $f(x)$.
5. The language $A_{NFA} = \{< N, x >: N \text{ is an NFA that accepts } x\}$.
   (a) Show that $A_{NFA} \in \mathbf{NL}$.
   (b) Show that A_{NFA} is **NL**-complete.