

School of Mathematical and Computational Sciences
Indian Association for the Cultivation of Science

*Master's/Integrated Master's-PhD Program/ Integrated
Bachelor's-Master's Program/PhD Course*

Theory of Computation II: COM 5108

Tutorial VI (21 September 2023)

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1. $5\text{-SUBSET-SSUM} = \{ \langle S, t \rangle : S = \{x_1, \dots, x_n : x_i \in \mathbb{N}\}, \exists S' = \{y_1, \dots, y_5\} \subseteq S, \sum_{i=1}^5 y_i = t \}$. Is 5-SUBSET-SSUM **NP**-complete?
2. $\text{PATH} = \{ \langle G, s, d \rangle : G \text{ is a graph where there is a path from } s \text{ to } d \}$. PATH is known to be in **P**. What is the repercussion if it is proved that PATH is not **NP**-complete.
3. Solve the following problems.

(i) What is the value of $n \left(\frac{1}{\ln n} \right)$ $n > 1$?

(ii) Show that $n \left(\frac{1}{c} \right)^{\log \log n}$ is bounded above (less than a constant), where $c \geq 3$ is a constant and $n \geq e$.

4. Prove that if there is a polynomial time reduction from SAT to SAT such that $\phi \mapsto \psi$ and $|\psi| = \sqrt[3]{|\phi|}$, then SAT is in **P**.
5. The language of undirected Hamiltonian path (UHAMPATH) is defined as follows.

$\text{UHAMPATH} = \{ \langle G, s, d \rangle : G \text{ is an undirected graph and there is a Hamiltonian path from } s \text{ to } d \}$.

- (i) Show that $\text{UHAMPATH} \in \text{NP}$.
- (ii) Show that UHAMPATH is **NP**-hard.