School of Mathematical and Computational Sciences

Indian Association for the Cultivation of Science

Master's/Integrated Master's-PhD Program/ Integrated Bachelor's-Master's Program/PhD Course

Theory of Computation II: COM 5108

Tutorial V (14 September 2023)

Instructor: Goutam Biswas *Autumn Semester 2023*

1. Consider the following languages and answer whether they belong to **P** or they are **NP**-complete?

   (i)
   $$SAT_e = \{\phi\ 0\ 1^{2^n} :\ \phi \in SAT \text{ and } |\phi| = n\}.$$

   (ii)
   $$SAT_p = \{\phi\ 0\ 1^{n^c} :\ \phi \in SAT,\ n = |\phi|, \text{ and } c \text{ is a constant}\}.$$

2. If $\mathbf{P} = \mathbf{NP}$ and $L \in \mathbf{P}$, but $L \neq \phi$ or Σ^*, then prove that L is **NP**-complete.

3. UNARY-SSUM $= \{< S, t >:\ S = \{x_1, \cdots, x_k : x_i \in \mathbb{N}$, where x_i's are represented as unary numerals $\}$ is a multiset and for some $\{y_1, \cdots, y_l\} \subseteq S,\ \sum_{i=1}^{l} = t\}$.

   (i) Is UNARY-SSUM in **NP**?

   (ii) Is UNARY-SSUM **NP**-hard?

4. In the proof of Cook-Levin theorem a window of size 2×3 was used to establish the correctness of transition from configuration C_i to C_{i+1}. Justify that it cannot be done using window of size 2×2.

5. Give a polynomial time reduction of 3COL to SAT. What is the time complexity of the reduction.