School of Mathematical and Computational Sciences

Indian Association for the Cultivation of Science

Master's/Integrated Master's-PhD Program/ Integrated Bachelor's-Master's Program/PhD Course

Theory of Computation II: COM 5108

Tutorial IV (07 September 2023)

Instructor: Goutam Biswas — *Autumn Semester 2023*

1. An *enumerator* Turing machine (ETM) is one that enumerates strings of a language. It does not take any input. Prove that a language is Turing recognizable if and only if it is enumerable by some ETM.

2. Let $L_1, L_2 \subseteq \Sigma^*$ be Turing recognizable languages.

   (a) Show that $L_1 \cap L_2$ is Turing recognizable.

   (b) Show that $L_1L_2 = \{x \in \Sigma^* : \exists u, v,\ x = uv,\ u \in L_1, v \in L_2\}$ is Turing recognizable.

   What is your conclusion?

3. Let $L_1, L_2 \subseteq \Sigma^*$ be in **P**.

   (a) Show that $L_1 \cup L_2$ is in **P**.

   (b) Show that L_1L_2 is in **P**.

   What is your conclusion?

4. Let $L_1, L_2 \subseteq \Sigma^*$ be in **NP**.

   (a) Show that $L_1 \cup L_2$ is in **NP**.

   (b) Show that L_1L_2 is in **NP**.

   What is your conclusion?

5. Consider the following algorithm where $a \in \mathbb{Z}_n$, n and E are positive integers. The binary representation of E is $(e_{k-1}e_{k-2}\cdots e_1e_0)$.

```
what(a, e, n)
    v ← 1
    s ← a mod n
    while e ≥ 1
        if (e mod 2) = 1 then v ← (v × s) mod n
        s ← s^2 mod n
        e ← e ÷ 2
    return v
```

   (a) What is the length of the input?

   (b) Compute `what(10, 10, 7)`.

   (c) What does it compute?

   (d) Analyze the time complexity.