School of Mathematical and Computational Sciences

Indian Association for the Cultivation of Science

Master's/Integrated Master's-PhD Program/ Integrated Bachelor's-Master's Program/PhD Course

Theory of Computation II: COM 5108

Tutorial II (17 August 2023)

Instructor: Goutam Biswas *Autumn Semester 2023*

1. What does the TM $M = (\{s, q_0, q_1\}, \{0, 1, \triangleright, \sqcup\}, \delta, s)$ compute on input $\triangleright x$, where $x \in \{0,1\}^+$?

$p \in Q$	$\sigma \in \Sigma$	$\delta(p, \sigma) = (q, \gamma, D)$
s	$\triangleright$	$(s, \triangleright, \rightarrow)$
s	0	$(s, 0, \rightarrow)$
s	1	$(s, 1, \rightarrow)$
s	$\sqcup$	$(q_0, \sqcup, \leftarrow)$
q_0	0	$(q_0, 0, \leftarrow)$
q_0	1	$(q_1, 1, \leftarrow)$
q_0	$\triangleright$	$(h, \triangleright, \rightarrow)$
q_1	0	$(q_1, 1, \leftarrow)$
q_1	1	$(q_1, 0, \leftarrow)$
q_1	$\triangleright$	$(h, \triangleright, \rightarrow)$

Ans. Computes the 2's complement of the given data.

2. (a) Design a single tape Turing machine that computes a function $f : \{0,1\}^* \rightarrow \{0,1\}^*$ i.e. it takes an input $x \in \{0,1\}^*$ and produces an output $f(x) = y \in \{0,1\}^*$ such that each '0' and '1' of x will be replaced by '00' and '11' respectively. As examples, $f(\varepsilon) = \varepsilon$, $f(0) = 00$, $f(1) = 11$, $f(101) = 110011$ etc.

   (b) Show every step of computation (sequence of configurations) on input $\varepsilon, 0, 10, 110$

   (c) Compute the number of steps in terms $|x| = n$ as accurately as you can.

   (d) What is the time complexity?

3. (a) Design a 2-tape Turing machine for the language of **(2a)**. The output will be on the second tape. Clearly specify the start and end configurations.

   (b) Show every step of computation on input $\varepsilon, 0, 10, 110$

   (c) Compute the number of steps in terms $|x| = n$ as accurately as you can.

   (d) What is the time complexity?